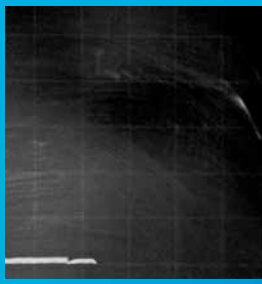
Titration Fundamentals



Titration
Compendium

Titration Fundamentals

for Education and Background Information

METTLER TOLEDO

Content

1 Introduction

A balance, a burette and a suitable chemical reaction suffice to solve many quantitative analytical problems. The analytical technique employed is called titration or titrimetric analysis (titrimetry). The expression "volumetric analysis" is not recommended [1]. In a titration, part of the sample containing the substance to be analyzed (the analyte) is dissolved in a suitable solvent. A second chemical compound, the titrant, is added as a solution of known concentration in a controlled manner until the analyte has reacted quantitatively. From the consumption and concentration of the titrant as well as the weight of sample used in the analysis, the content of the analyte can be calculated.

From the above definition it follows that the following requirements must be fulfilled before a titration can be performed:

- The basic chemical reaction – the titration reaction – must be rapid, straightforward and quantitative.
- It must be possible to either prepare a titrant of exactly known content or determine the reacting strength (titer) of the solution accurately.
- The course of the titration must be observable. The method used to follow the titration progress is called indication.
- Determination of the equivalence point – the point at which the number of entities (equivalents) of the titrant added is the same as the number of entities of sample analyte present – must be unambiguous.

The titration reaction, the indication, the control and evaluation of the titration as well as the assessment of the results (statistics) form the focal points of this monograph.

[1] IUPAC Compendium of Analytical Nomenclature, Pergamon Press, 1978, page 42, see also www.iupac.org/Publications

2 Base units of titrimetric analysis

The base units and calculation parameters of titrimetric analysis are associated with the base quantity amount of substance and its base unit mole of the international system of units (SI) [1]. The concepts and definitions regarding amount of substance and the quantities derived from it are defined in [2].

Mole
The SI base unit for amount of substance is the mole (symbol of unit: mol). The mole is the amount of substance of a system that contains just as many elementary entities as there are atoms in 12 g of the carbon ^{12}C isotope. One mole of a substance contains $6.022 \cdot 10^{23}$ elementary entities. These can be atoms, molecules, ions, groups of atoms or electrons.

Amount of substance specification
The entities referred to in specifications of the amount of substance should be entered in brackets after the amount of substance symbol n.

Examples:
n(HCl) = 2 mol
$n(Ca^{2+})$ = 4 mmol

Molar mass M
The molar mass (symbol M) is a quantity related to the amount of substance.
The molar mass of a substance X is defined as its mass m divided by amount of substance n(X).

$$M(X) = \frac{m}{n(X)}$$

The usual unit in analysis is g/mol.

Examples:
M(NaOH) = 39.997 g/mol
M(EDTA) = 372.24 g/mol

Amount-of-substance concentration c(X)
The amount-of-substance concentration of a solution of an entity X (symbol c(X)) is the amount of substance n(x) divided by the volume V of the solution.

$$c(X) = \frac{n(X)}{V}$$

The usual units employed in analysis are mol/L and mmol/L.

Examples:
c(HCl) = 0.1 mol/L
c(AgCl) = 0.01 mol/L

Notes:
The simpler designation "concentration" for amount-of-substance concentration is allowed.

The old designation "molarity" is no longer used.

Titer t
The titer (symbol t) of a titrimetric solution is the quotient of the actual concentration (ACTUAL value) and the expected concentration (NOMINAL value).

$$t = \frac{c(X, \text{ACTUAL})}{c(X, \text{NOMINAL})}$$

Examples:
c(HCl, ACTUAL) = 0.1036 mol/L
c(HCl, NOMINAL) = 0.1 mol/L

The titer t = 1.036.

Note:
The name "factor" for the titer is no longer used.

Equivalent, Equivalent number z*

In the previous examples the amount of substance n, the molar mass M and the amount-of-substance concentration c refer to whole entities. In titrimetric analysis, reference to fractions of such entities is often more suitable.

The equivalent entity, abbreviated to equivalent, is the fraction 1/z* of such an entity. The number of equivalents z* of each entity X is called the equivalent number.

Examples of equivalent numbers:

1. Neutralization (acid-base) equivalent: In a neutralization reaction, the entity X combines with or releases z* protons.

a. HCl:
z* = 1
$H^+ + Cl^- + Na^+ + OH^- \rightarrow Na^+ + Cl^- + H_2O$

b. H_2SO_4:
z* = 2
$2H^+ + SO_4^{2-} + 2\ Na^+ + 2\ OH^- \rightarrow 2\ Na^+ + SO_4^{2-} + 2\ H_2O$

2. Redox equivalent: In a redox reaction, the reaction partners change their oxidation state.

a. $KMnO_4/Fe^{2+}$:
$KMnO_4$: z* = 5
Fe^{2+}: z* = 1

$$\overset{\text{VII}}{K^+} + \overset{\text{II}}{MnO_4^-} + 5\overset{\text{II}}{Fe^{2+}} + 8H^+ \rightarrow K^+ + \overset{\text{II}}{Mn^{2+}} + 5\overset{\text{III}}{Fe^{3+}} + 4H_2O$$

b. $KMnO_4/Mn^{2+}$:
$KMnO_4$: z* = 3
Mn^{2+}: z* = 2

$$2\ K^+ + 2\ \overset{\text{VII}}{MnO_4^-} + 3\ \overset{\text{II}}{Mn^{2+}} + 4\ OH^- \rightarrow 2\ K^+ + 5\ \overset{\text{IV}}{MnO_2} + 2\ H_2O$$

Note:
The old name "valency" is no longer used.

In titrimetric analysis the following quantities are related to equivalents:

- amount of substance
- molar mass
- concentration.

Amount of substance of equivalents

The amount of substance n of an equivalent of entity X (symbol n(1/z* X)) is equal to the product of the equivalent number z* and the amount of substance n of entity X.

$$n(\frac{1}{z^*}X) = z^* \cdot n(X)$$

The common units are mol and mmol.

Examples:

$n(1/2\ Ca^{2+})$ = 2 mmol

$n(1/5\ KMnO_4)$ = 5 mol

Molar mass of equivalents

The molar mass M of an equivalent of entity X (symbol M(1/z* X)) is the molar mass M of the entity X divided by the equivalent number z*.

$$M(\frac{1}{z^*}X) = \frac{M(X)}{z^*}$$

The unit is g/mol.

Examples:

$M(1/2\ H_2SO_4)$ = 49.04 g/mol

$M(1/5\ KMnO_4)$ = 31.61 g/mol

$M(1/3\ KMnO_4)$ = 52.68 g/mol

$M(1/6\ K_2Cr_2O_7)$ = 49.03 g/mol

Amount-of-substance concentration of equivalents (equivalent concentration)
The amount-of-substance concentration of a solution of an equivalent of entity X (symbol c(1/z* X)) is the amount of substance n(1/z*X) of an equivalent of X divided by the volume V of the solution.

$$c(\frac{1}{z^*}X) = \frac{n(\frac{1}{z^*}X)}{V} = \frac{m}{M(\frac{1}{z^*}X) \cdot V} = \frac{m \cdot z^*}{M(X) \cdot V}$$

The usual units are mol/L and mmol/L.

The following relation holds:

$$c(\frac{1}{z^*}X) = z^* \cdot c(X)$$

Example:
How large is the amount-of-substance concentration c(1/6 $K_2Cr_2O_7$) of 1 g $K_2Cr_2O_7$ in 50 mL water?

$$c(\frac{1}{6}K_2Cr_2O_7) = \frac{1 \cdot 6}{294.185 \cdot 0.05} = 0.408 \text{ mol/L}$$

Note:
The amount-of-substance concentration replaces the concepts molarity, normality, Val/L as well as the concepts "molar", "M", "normal", "N", etc. derived from them. These are no longer used. Unless the reaction equation is specified, the old descriptions do not allow explicit recognition of the equivalent; for example 0.1N $KMnO_4$ could apply equally well to 1/3 $KMnO_4$ and 1/5 $KMnO_4$. In contrast, specification of the amount-of-substance concentration c(1/5 $KMnO_4$) = 0.1 mol/L is unambiguous.

Concentration of a titrant

The concentration of a titrant should be specified as equivalent concentration.

Example:
$c(1/2\ H_2SO_4) = 0.1$ mol/L

The amount-of-substance concentration needed for preparation of a titrimetric solution of the equivalent concentration c(1/z* X) is calculated with the aid of the formula

$$m = \frac{c(\frac{1}{z^*}X) \cdot M(X) \cdot V}{z^*}$$

Example:
Preparation of 100 mL of a titrimetric solution of sulfuric acid of concentration $c(1/2\ H_2SO_4) = 0.1$ mol/L.

Amount of substance required

$$m = \frac{0.1 \cdot 98.08 \cdot 0.1}{2} = 0.4904 \text{ g}$$

[1] Bureau International des Poids et Mesures, Le Système International d'Unités (SI), 5th French and English Edition, BIPM, Sèvres 1985

[2] IUPAC Compendium of Analytical Nomenclature, Pergamon Press, 1978, page 175 ff. See also DIN 32625

3 The titration reaction

The basis of each titrimetric method of analysis is the chemical reaction of the analyte with the titrant. In order to understand the demands on this titration reaction a brief introduction into the fundamentals of the thermodynamics of chemical reactions is called for.

3.1 Thermodynamic fundamentals

3.1.1 The law of mass action

Every reversible chemical reaction proceeds to an exactly defined equilibrium condition. It is characterized by the following general equation:

$$aA + bB + cC \underset{v_2}{\overset{v_1}{\rightleftarrows}} xX + yY + zZ$$

In this equation a, b, c, x, y and z represent the number of moles of the substances A, B, C, X, Y and Z participating in the reaction in stoichiometric proportion. At equilibrium, the rates of the forward and back reactions are equal ($v_1 = v_2$). This equilibrium is described by the so-called law of mass action.

$$K = \frac{[X]^x \cdot [Y]^y \cdot [Z]^z}{[A]^a \cdot [B]^b \cdot [C]^c}$$

The constant K is known as the thermodynamic equilibrium constant.

The concentrations of entities X, Y and Z are designated here by [X], [Y] and [Z] ([X] = c(X)). This notation is usual in analytical chemistry.

Strictly speaking, the law of mass action can not be applied directly to the analytical concentrations of the reaction partners. Real chemical systems are distinguished by the mutual interaction of all molecules present. In solution, interactions occur between the molecules of the dissolved substance and the solvent molecules. Here, it is only the quasi-free or "effective" concentrations of the substances participating in reaction, the so-called activities, that are decisive for the chemical reaction and the law of mass action. But for a formal understanding and the calculations, only analytical concentrations are considered in the present discussion.

The demand that the titration reaction proceed quantitatively and to completion is fulfilled when the equilibrium constant K is so large that the equilibrium concentration of the analyte is infinitely small in comparison with its concentration before titration.

The equilibrium constant K provides no direct information regarding the rate of the titration reaction. Decisive for this is the rate of the forward reaction v_1, the reaction of the analyte with the titrant.

The following sections treat the thermodynamic constants that appear most frequently in potentiometry, the solubility product of sparingly soluble salts, the ionic product of water and the acidity constant of weak acids.

3.1.2 The solubility product of sparingly soluble salts

Many salts are only slightly soluble. If solutions of the corresponding ions are mixed, precipitates are formed. The processes at the surface of a salt in contact with a saturated solution lead to the establishment of a heterogenous equilibrium. Ions from the salt constantly pass into solution, and ions from the solution are incorporated in the salt lattice.

$$AB(solid) \rightleftarrows A^+ + B^-$$

For this equilibrium the following law of mass action applies:

$$K = \frac{[A^+] \cdot [B^-]}{[AB]}$$

As long as solid salt AB is present as precipitate, the concentration of AB remains constant and is thus included in the equilibrium constant. This gives rise to the solubility product K_{sp}:

$$K_{sp} = [A^+] \cdot [B^-]$$

A sparingly soluble salt is always precipitated when the solubility product of the participating ions is exceeded. The lower the solubility product, the more insoluble the salt.

The solubility product of salts having the general formula AB_2 has the following form:

$$K_{sp} = [A^{2+}] \cdot [B^-]^2$$

The solubility product of many salts shows a large temperature dependence:

Examples of solubility products (type AB):

Salt	K_{sp} [mol²/L²]
AgCl	10^{-10}
AgBr	$5 \cdot 10^{-13}$
AgI	10^{-16}
$PbSO_4$	10^{-8}

3.1.3 The ionic product of water

When the conductivity of water is examined using very sensitive instruments, it is apparent that even ultrapure, repeatedly distilled water has a very low conductivity. It is due to the following reaction:

$$H_2O + H_2O \rightleftarrows H_3O^+ + OH^-$$

The forward reaction describes the proton transfer from one water molecule to another. This equilibrium is present not only in pure water but also in all aqueous solutions. The corresponding law of mass action is:

$$K = \frac{[H_3O^+] \cdot [OH^-]}{[H_2O]^2}$$

The concentrations of the H_3O^+ and OH^--ions in solution can be changed drastically by addition of an acid or base. However the concentration of the H_2O molecules (55.5 mol/L) remains constant in dilute solutions. The law of mass action can thus be simplified:

$$K_w = [H_3O^+] \cdot [OH^-]$$

The equilibrium constant K_w is known as the ionic product of water. It depends on the temperature and is 10^{-14} mol²/L² at 23 °C.

In dilute aqueous solutions the product of $[H_3O^+]$ and $[OH^-]$ is thus constant. If one of the two concentrations is known, the other can be calculated from a knowledge of K_w. In a neutral solution the concentrations $[H_3O^+]$ and $[OH^-]$ are equal:

$$[H_3O^+] = [OH^-] = (K_w)^{1/2} = 10^{-7} \text{ mol/L}$$

If, for example, the H_3O^+ concentration is increased to 10^{-2} mol/L by addition of acid, the OH^- concentration decreases to 10^{-12} mol/L. Specification of one of these concentrations allows unequivocal identification of the nature of an aqueous solution. This led to the introduction of the pH concept as

$$pH = - \log \left[H_3O^+\right]$$

In acidic solutions ($[H_3O^+] > 10^{-7}$) the pH is less than 7, whereas in alkaline solutions it is greater than 7. The pH of a neutral aqueous is 7.

3.1.4 The strength of acids and bases

The reaction of weak acids with water is described by the following equilibrium:

$$HA + H_2O \rightleftarrows H_3O^+ + A^-$$

Acid HA reacts with the base H_2O to form the conjugate base A^- of HA and the conjugate acid of H_2O, namely H_3O^+.

The corresponding law of mass action is:

$$K = \frac{\left[H_3O^+\right] \cdot \left[A^-\right]}{\left[HA\right] \cdot \left[H_2O\right]}$$

In dilute solutions ($[H_2O]$ = constant) the following formula applies:

$$K_a = \frac{\left[H_3O^+\right] \cdot \left[A^-\right]}{\left[HA\right]}$$

The equilibrium constant K_a is known as the acid dissociation constant of acid HA that characterizes the strength of an acid. Strong acids have a large acidity constant, weak acids a very low one. The negative logarithm of K_a is frequently employed in calculations:

$$pK_a = -\log K_a$$

Examples of pK_a values of a few acid-base pairs (25 °C):

Acid	Base	pK_a
$HClO_4$	ClO_4^-	-9
HCl	Cl^-	-6
H_2SO_4	HSO_4^-	-3
HSO_4^-	SO_4^{2-}	1.96
H_3PO_4	$H_2PO_4^-$	1.96
CH_3COOH	CH_3COO^-	4.75
$H_2PO_4^-$	HPO_4^{2-}	7.21
NH_4^+	NH_3	9.21
HPO_4^{2-}	PO_4^{3-}	12.32

The reaction of the base A^- with water can be described in an analogous manner:

$$A^- + H_2O \rightleftarrows HA + OH^-$$

The corresponding basicity constant K_b follows from the law of mass action:

$$K_b = \frac{[HA] \cdot [OH^-]}{[A^-]}$$

For a conjugate acid-base pair HA/A^- follows:

$$K_a \cdot K_b = [H_3O^+] \cdot [OH^-] = K_w$$

Polyprotic acids or polyequivalent bases that donate (accept) protons in steps have a separate acidity (basicity) constant for each ionization step.

Example:

$H_3PO_4 + H_2O \rightleftarrows H_2PO_4^- + H_3O^+ \quad pK_a = 1.96$

$H_2PO_4^- + H_2O \rightleftarrows HPO_4^{2-} + H_3O^+ \quad pK_a = 7.21$

$HPO_4^{2-} + H_2O \rightleftarrows PO_4^{3-} + H_3O^+ \quad pK_a = 12.32$

3.2 The most important titration reactions

This section contains a summary of the titration reactions important in titration practice.

3.2.1 Acid-base titrations in aqueous solutions

In the titration of an acid HA with a strong base (e.g. NaOH) the following two chemical equilibria occur:

$$HA + H_2O \rightleftarrows H_3O^+ + A^-$$

$$K_a = \frac{[H_3O^+] \cdot [A^-]}{[HA]}$$

$$2\ H_2O \rightleftarrows H_3O^+ + OH^-$$

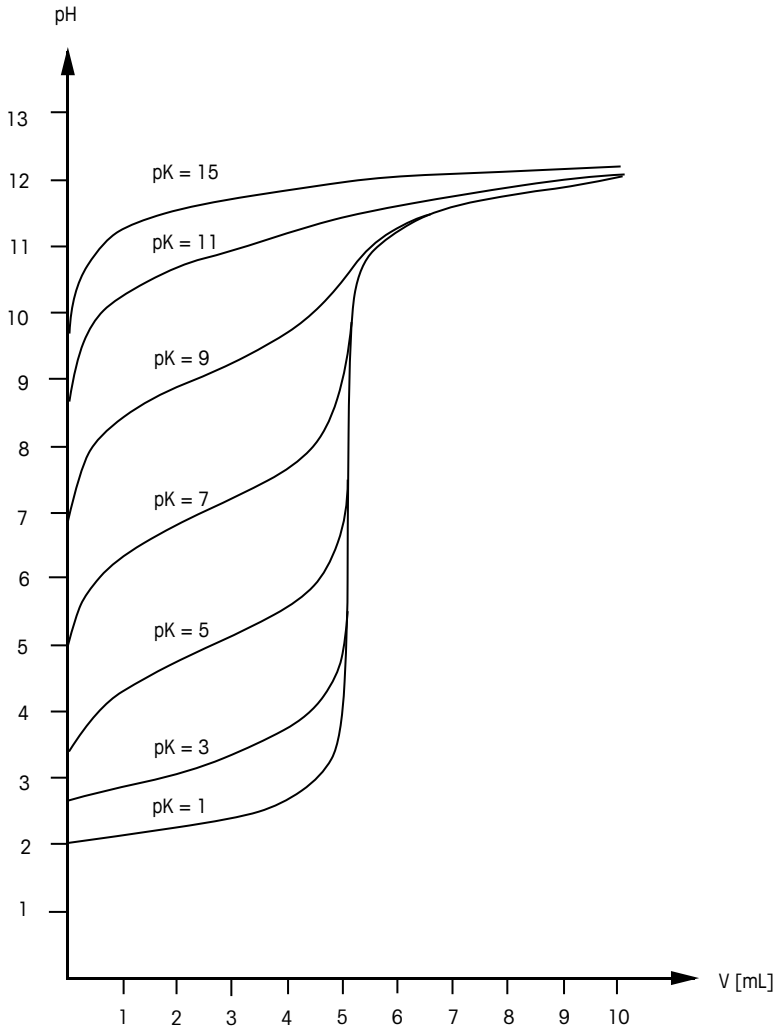
$$K_w = [H_3O^+] \cdot [OH^-]$$

Acid-base reactions are very fast, and the chemical equilibrium is established extremely rapidly. Acid-base reactions in aqueous solutions are thus ideal for titrations. If the solutions used are not too dilute, the shape of the titration curves depends only on the acidity constant K_a as the following figure shows.

Notes:

- Very weak acids are difficult to titrate in aqueous solution. In the figure below it can be seen that for pK_a values greater than 10, the corresponding titration curve no longer exhibits any jump in the region of the equivalence point.
- Bases can be titrated with a strong acid in an analogous manner. The same titration curves result if K_a is substituted by K_b and pH by pOH ($pH + pOH = pK_w = 14$).
- Polyprotic acids (e.g. the first two ionization steps of phosphoric acid) and mixtures of acids can easily be titrated separately if the acidity constants differ by at least two pK units.

Titrations of 50 mL 0.01 mol/L of different HA acids with 0.1 mol/L NaOH yield the following titration curves:



3.2.2 Acid-base titrations in nonaqueous solution

Titrations can also be performed in nonaqueous solvents ([1], [2], [3], [4]).
The use of nonaqueous solvents is advantageous under the following conditions:

- The analyte is only sparingly soluble in water.
- The analyte or the titrant enter into an undesired reaction with water (e.g. acid chloride, acid anhydride).
- A mixture of analytes is present; this cannot be analyzed selectively in aqueous solution (pK_a values too close together).
- The analyte is too weak an acid or base in water.

The main applications in nonaqueous media are acid-base titrations.

Like water, each suitable solvent HS for acid-base titrations acts both as an acid and a base:

Acid:

$$HS \rightleftarrows H^+ + S^-$$

$$K_a^{HS} = \frac{[H^+] \cdot [S^-]}{[HS]}$$

Base:

$$HS + H^+ \rightleftarrows SH_2^+$$

$$K_b^{HS} = \frac{[SH_2^+]}{[HS] \cdot [H^+]}$$

The sum of the above two equilibria gives the autoprotolysis constant K_s^{HS} of the medium:

$$2\ HS \rightleftarrows SH_2^+ + S^-$$

$$K_S^{HS} = [SH_2^+] \cdot [S^-] = K_a^{HS} \cdot K_b^{HS}$$

The solvent is thus characterized by the acidity constant K_a^{HS}, the basicity constant K_b^{HS} and the autoprotolysis constant K_s^{HS}.

The following rules of thumb apply to the use of nonaqueous solvents:

- If the acid HA to be determined is very weak, the acidic behavior of the solvent must be less pronounced than that of water (small K_a^{HS}).

- If the base B to be determined is very weak, the basic behavior of the solvent must be less pronounced than that of water (small K_b^{HS}).
- The smaller the autoprotolysis constant K_s^{HS}, the greater the potential jump at the equivalence point.
- Many nonaqueous solvents show so-called differentiating (nonleveling) properties that allow for selective determination of substances having similar pK_a values in water.

Nonaqueous media exhibit several peculiarities that should be noted:

- The coefficient of expansion of organic solvents is considerably larger than that of water. The temperature dependence of the titer can thus be very large (up to 0.2% for a temperature change of 1 °C).
- Many nonaqueous solvents are more volatile than water and are sensitive to CO_2. It is thus essential to check the titer frequently.

Examples of titrants in nonaqueous solvents:
Acids: HCl in isopropanol, perchloric acid in glacial acetic acid
Bases: KOH in ethanol, sodium methoxide in chlorobenzene.

3.2.3 Precipitation titrations

Precipitation titrations are distinguished by the formation of a sparingly soluble reaction product (precipitate) between the titrant and the analyte. The classic example is the determination of halogenides with silver nitrate:

$$Ag^+ + X^- \rightleftarrows AgX(solid)$$

$$K_L = [Ag^+] \cdot [X^-]$$

$$X^- = Cl^-, Br^-, I^-$$

Precipitation titrations are characterized by the following points:

- The titration reaction may be quite slow under certain circumstances.
- At the start of the titration, the solution may become supersaturated before a precipitate is formed.
- With solutions that are too concentrated, inclusions of sample and/or titrant may occur in the precipitating solid, thereby falsifying the result. An effective countermeasure is rapid stirring and also the addition of polyvinyl alcohol (PVA) as emulsifier. This yields a fine dispersion of the precipitate.

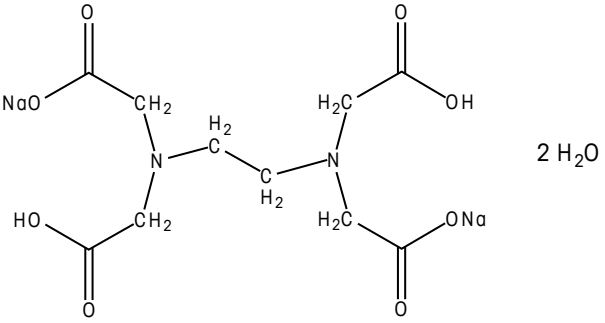
3.2.4 Complexometry

Complexometric methods allow the titration of a large number of metal ions. A typical example is the formation of chelates between the metal ion M and the complexing agent Y.

$$M + Y \rightleftarrows MY$$

$$K_{MY} = \frac{[MY]}{[M] \cdot [Y]}$$

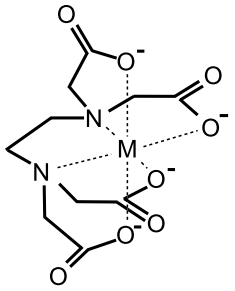
The most important complexing agent is the disodium salt of ethylenediaminetetraacetic acid (EDTA: abbreviation $Na_2H_2Y \cdot 2H_2O$, molar mass: 327.24 g/mol):



All complexes contain metal and ligand in a 1:1 ratio, irrespective of the charge of the metal ion.

$$M^{2+} + H_2Y^{2-} \rightarrow MY^{2-} + 2H^+$$

$$M^{3+} + H_2Y^{2-} \rightarrow MY^{-} + 2H^+$$



If these reactions run in unbuffered solutions, the pH is lowered. If a change in pH has to be avoided, substances with sufficient buffer capacity must be added. In alkaline solutions the metal is more tightly bound in the complex than in acidic solutions.

Among the applications of the complexometric titration, the determination of water hardness (Ca, Mg) has achieved the greatest importance.

3.2.5 Redox titrations

If two reaction partners can be interconverted by the gain or loss of electrons, a redox system is present. The process underlying this chemical reaction is called a redox reaction (= electron shift). The two partners are known as a conjugate redox couple.

Example:
$Fe^{3+} + e^- \rightleftarrows Fe^{2+}$

One of these entities gives up electrons. This process is called oxidation. The other entity gains these released electrons. This process is known as reduction.

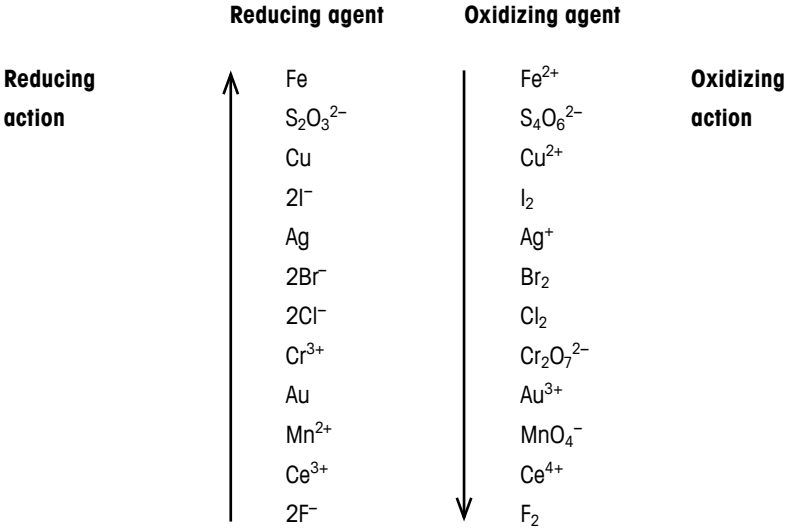
Substances that can oxidize other substances are called oxidizing agents. Substances that can reduce other substances are known as reducing agents.

Since electrons never occur in the free state in perceptible concentration, oxidation and reduction reactions can only occur together. One reaction releases exactly the same number of electrons as the other reaction requires. There must thus always be two reaction couples participating in a redox reaction.

$Red_1 + Ox_2 \rightarrow Ox_1 + Red_2$

Examples:
$Fe + Cu^{2+} \rightarrow Fe^{2+} + Cu$
$2I^- + Br_2 \rightarrow I_2 + 2Br^-$

Through a comparison of a number of such reactions, the strength of oxidizing or reducing agents can easily be defined qualitatively. Similar to acids (bases), reducing and oxidizing agents can also be arranged in a series (redox series).



This redox series shows a representative selection of redox couples that includes not only the most well-known titrants but also several metals and the halogens. It can be seen immediately from this chart that, for example, metallic iron in copper(II) solutions will be oxidized.

Examples of important redox reactions for titration:

Manganometry

Manganometry is based on the powerful oxidizing effect of potassium permanganate. The overwhelming number of redox titrations with $KMnO_4$ are performed in sulphuric acid solutions according to the following scheme:

$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$

Manganese with oxidation number +7 is reduced to Mn^{2+}.

Example:
Determination of peroxides

$2MnO_4^- + 5H_2O_2 + 6H^+ \rightarrow 2Mn^{2+} + 5O_2 + 8H_2O$

Iodimetry/Iodometry

One of the most important redox couples is iodide/iodine. The fundamental process

$I_2 + 2e^- \rightleftarrows 2I^-$

is completely reversible. There are thus always two possibilities:

1. Reduction of iodine (Iodimetry):

$I_2 + 2e^- \rightarrow 2I^-$

In this manner reducing agents can be determined directly with iodine solution as titrant.

Example:
Determination of SO_2

$SO_2 + I_2 + 2H_2O \rightarrow H_2SO_4 + 2HI$

2. Oxidation of iodide (Iodometry):

$2I^- \rightarrow I_2 + 2e^-$

The determination of oxidizing agents in iodometry is performed as a replacement titration in the majority of cases (see section 5). An excess of iodide is added to the sample.

Example:
Determination of copper $2Cu^{2+} + 2I^- \rightarrow 2Cu^+ + I_2$

The liberated iodine is titrated with a suitable reducing agent. Sodium thiosulfate is used almost exclusively today.

$2S_2O_3^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$

Dichromate method

Chromium with oxidation state +6 is reduced by a large number of reducing agents in acidic solution. Use is made of this property in the cleaning of glass vessels with chromosulfuric acid. The dichromate ion $Cr_2O_7^{2-}$ is stable in acidic solution but can be reduced to the chromium ion Cr^{3+} in the presence of hydrogen ions by gain of six electrons (three per chromium(VI)) according to the equation

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

The hydrogen ions are consumed with formation of water.

Dichromate as a titrant has gained practical importance in the determination of the chemical oxygen demand (COD) in waste water analysis [5]. The COD determination is based on the oxidation of organic compounds with chromosulfuric acid using silver sulfate catalyst.

Cerimetry

Cerium(IV) sulfate is a powerful oxidizing agent. The oxidation state of cerium changes only by one:

$$Ce^{4+} + e^- \rightarrow Ce^{3+}$$

The cerium(IV) sulfate solution (prepared in H_2SO_4) has a stable titer and is insensitive to both light and heat. In contrast to permanganate solutions it can also be used for titrations in highly concentrated hydrochloric acid solution [6]. It is thus extremely versatile.

Diazotizations and nitrosations

One important oxidizing agent is sodium nitrite. It allows the determination of primary amines through diazotization, and the determination of secondary amines and phenols through nitrosation in acidic solution.

Simplified reaction schemes:

Primary amines:

$$R - NH_2 + NO_2^- + 2H^+ \rightarrow [R - N = N]^+ + 2H_2O$$

Secondary amines:

$$R - NH - R + NO_2^- + H^+ \rightarrow R - \underset{\displaystyle NO}{\underset{|}{N}} - R + H_2O$$

[1] W. Huber, Titrations in nonaqueous solvents, New York: Academic Press, 1967
[2] J. Kucharsky and L. Safarik, Titrations in nonaqueous solvents, Elsevier Publishing Company, Amsterdam, 1965
[3] K. Stammbach, Titrationen in nichtwässrigen Lösungsmitteln, Schweizerische Laboranten-Zeitschrift, CH-4127 Birsfelden, offprint 1970
[4] I. Gyenes, Titrations in non-aqueous media, London: Iliffe, 1967
[5] DIN standard 38 409 – H 41-2 (1980)
[6] Permanganate oxidizes Chloride to elemental Chlorine.

4 Indication methods

The progress of the titration, the chemical reaction and the determination of the end point must be observable. Traditionally, the titration was observed visually, usually by addition of color indicators to the solution as only a few reactions are self-indicating (e.g. reactions with iodine and permanganate).

Over the years many disadvantages have led to the replacement of visual indication by electrochemical and photometric indication. A major advantage is that these techniques can be automated.

- only the end point and not the complete titration profile is indicated,
- recognition of the color by human eye is not objective,
- many titrations cannot be indicated visually,
- with color indicators an arbitrary end point of the titration is defined that does not coincide with the equivalence point,
- the color indicator is also titrated and this distorts the result and
- the cost of the chemicals and sample pretreatment is usually greater than in indication using an electrochemical sensor

With electrochemical sensors, namely electrodes, charge transfers and charge separations that arise at phase boundary surfaces can be determined (potentiometry) or generated and altered by means of an imposed current (voltametry, amperometry).

With photometric sensors the decrease in intensity of a light beam passing through the sample can be measured at a specified wavelength.

4.1 Electrochemical indication

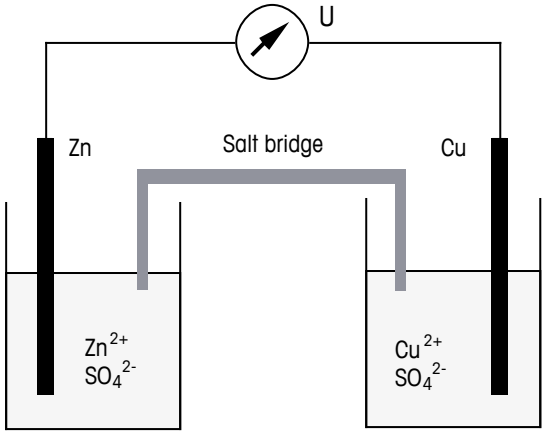
The processes that occur at an electrode in a galvanic cell form the basis of electrochemical indication methods.

4.1.1 Galvanic cells

A galvanic cell comprises two electrodes and one solution or two solutions separated from each other but connected by an electrically conducting salt bridge (= half cells). Such an arrangement generates electrical energy through electrochemical processes. Galvanic cells are also popularly referred to as batteries.

An oxidation takes place at one of the electrodes and a reduction at the other. The electrons released in the oxidation process in the first half cell are transported across the external bridge to the other electrode where reduction occurs. There is thus a potential difference between the two electrodes.

A simple galvanic cell can be demonstrated using the example of the reaction of metallic zinc and copper ions.

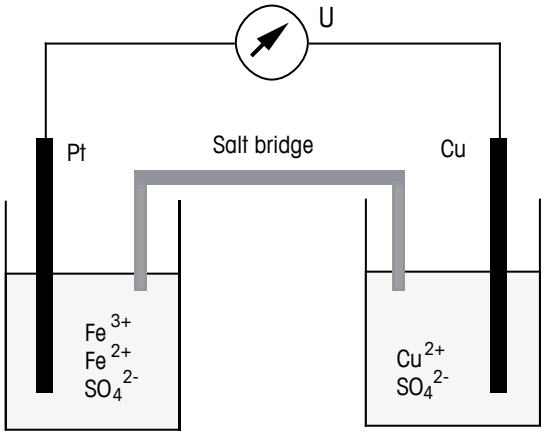


The following redox reactions take place:

$$\begin{array}{rcll} Zn & \rightarrow & Zn^{2+} + 2e^- & \textbf{Oxidation} \\ 2e^- + Cu^{2+} & \rightarrow & Cu & \textbf{Reduction} \\ \hline Zn + Cu^{2+} & \rightarrow & Zn^{2+} + Cu & \end{array}$$

The voltage (potential difference) measurable at the voltmeter, the slow dissolution of the zinc rod and the deposition of copper on the copper rod are proof of the electrochemical process.

Considerably more important are inert electrodes (e.g. platinum) that are not changed by the redox reaction:



In this galvanic cell the following reactions occur:

$$\begin{array}{rcll} Cu & \rightarrow & Cu^{2+} + 2e^- & \textbf{Oxidation} \\ 2e^- + 2Fe^{3+} & \rightarrow & 2Fe^{2+} & \textbf{Reduction} \\ \hline Cu + 2Fe^{3+} & \rightarrow & Cu^{2+} + 2Fe^{2+} & \end{array}$$

In the right half cell the copper rod slowly dissolves. At the platinum electrode in the left half cell Fe^{3+} ions take up electrons and are reduced to Fe^{2+} ions. The platinum rod itself remains unchanged and is thus referred to as an inert electrode.

It is impossible to measure the potential of a single electrode directly; only the difference in potential of two electrodes is accessible. The resulting potential of such an electrode assembly E_{tot} (so-called electromotive force) is given by the difference between the potentials E_1 and E_2 of the two electrodes:

$$E_{tot} = E_1 - E_2$$

The potential of a single electrode depends on the ionic concentration of the solution used to complete the half cell. This dependency is described by the Nernst equation:

$$E = E_0 + \frac{R \cdot T \cdot \ln 10}{n \cdot F} \cdot \log \frac{[Ox]}{[Red]}$$

with:

E_0: standard potential ([Ox]/[Red] = 1) of the electrode
R: molar gas constant
T: temperature (in K)
n: number of electrons transferred in the electrode reaction and
F: Faraday constant.

[Ox] and [Red] are the concentrations of the oxidized and reduced ionic species participating in the reaction.

At 25 °C the following equation is obtained:

$$E(mV) = E_0 + \frac{59.16}{n} \cdot \log \frac{[Ox]}{[Red]}$$

A change in the concentration ratio by a factor of ten causes a change in the electrode potential by 59.16/n mV.

The half cells with zinc and copper rods mentioned in the above examples are so-called electrodes of the 1st kind. Each metal that is immersed in a solution of one of its salts and can develop a reversible potential is called an electrode of the 1st kind.

A further example is the Ag/Ag^+ system. For the electrode reaction

$$Ag \rightleftarrows Ag^+ + e^-$$

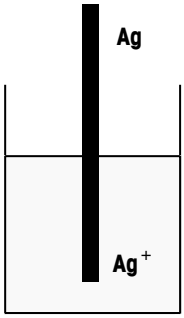
the Nernst equation applies

$$E = E_0 + \frac{R \cdot T \cdot \ln 10}{F} \cdot \log\left[Ag^+\right]$$

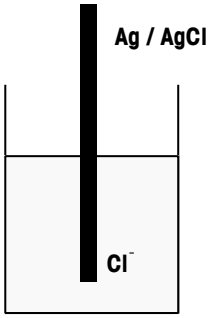
The potential of this half cell depends only on the silver ion concentration $[Ag^+]$ in the solution.

A metal that is coated with a layer of one of its sparingly soluble salts and immersed in a solution that contains the anion of the coating is called an electrode of the 2nd kind.

An example is a silver rod coated with silver chloride immersed in a chloride solution.



Electrode of the 1st kind



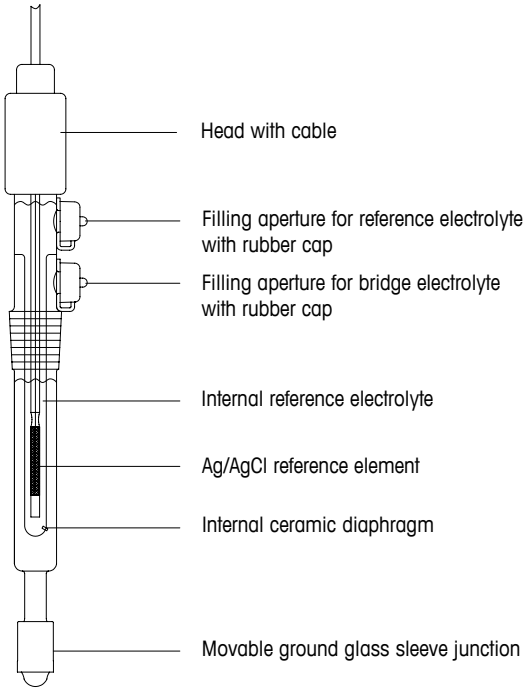
Electrode of the 2nd kind

Electrodes of the 2nd kind are very important as reference systems of reference electrodes.

A typical experimental setup in titration comprises one sensing electrode and one reference electrode. The task of the sensing electrode is to record all changes in the composition of the solution. The reference electrode must be capable of supplying a stable reference potential that is independent of these changes.

4.1.2 Reference electrodes

The nature of the reference system, the junction and the reference electrolyte determines the properties of the reference electrode. The reference electrode mainly used today (see illustration) is an electrode of the 2nd kind described above, with the reference system Ag/AgCl.



The reference system is in the form of a cartridge and contains an ample supply of silver and silver chloride. The cartridge is connected to the reference electrolyte (e.g. KCl: c(KCl) = 3 mol/L) via an internal junction.

The external junction ensures electrical contact between the reference electrode and the analysis solution. It must fulfill the following requirements:
- chemically inert
- low outflow rate of reference electrolyte at low electrical resistance
- no ion exchanger properties.

In addition to fine-pored ceramic junctions, sleeve junctions made of glass or plastic are used.

The following demands are made on reference electrolytes:
- constant chloride ion activity
- low electrical resistance
- chemically inert and neutral
- no reaction with analysis solution
- same mobility of cation and anion.

A concentrated solution of KCl fulfills virtually all these conditions.

To avoid a reaction of the reference electrolyte with constituents of the sample or titrant (e.g. Cl^- with Ag^+), double junction reference electrodes comprising two electrodes of the 2nd kind are often used.

In the DGi112-Pro is a double junction pH sensor for pH measurements and titrations in difficult matrices such as acid mixtures in electroplating baths.

In routine analysis, combined sensors have gained wide acceptance. Here, the sensing and reference electrodes are integrated in the same shaft (glass or plastic) (see section 4.1.4).

4.1.3 Metal sensors

Metal sensors, usually manufactured as electrodes of the 1st kind, are widely used in titration.

Sensors of the noble metals platinum and gold are used as redox sensors. They are eminently suitable for the indication of redox titrations.

In addition to measurement of the silver ion concentration (silver ion activity), metal sensors made of silver can be used for the indication of precipitation titrations (determination of halogens).

4.1.4 Glass sensors

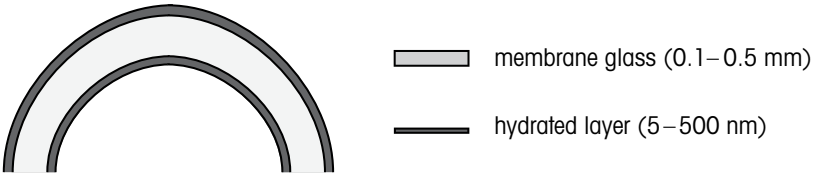
The glass sensor is the most important and most widely used sensor in analysis.

Glass membranes of composition

$SiO_2 - CaO - Na_2O$ or $SiO_2 - BaO - Li_2O$

that are in contact on both sides with a solution containing H^+ ions develop an electrical potential that depends on the difference in pH value of the boundary solutions. This phenomenon is based on the following physicochemical processes:

Each glass membrane of a pH sensor reacts with water to form a hydrated, gel-like layer (see figure). This hydrated layer is not visible since it has a thickness of only 5–500 nm, but it is of fundamental importance to the operating principle of the glass sensor.



The basis of the glass membrane is a three dimensional network of silicon and oxygen atoms with each silicon atom being surrounded by four oxygen atoms and each oxygen atom by two silicon atoms. The interstitial spaces in this irregular network are occupied by cations to ensure electroneutrality of the glass membrane.

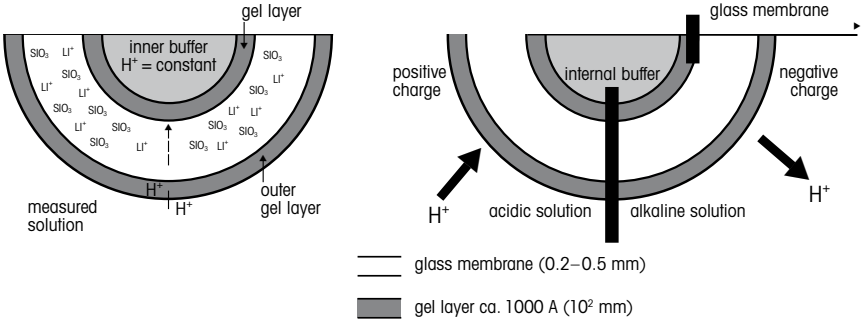
In the formation of the hydrated layer the following process occurs:

$$-Si-OM + H_2O \rightarrow -Si-OH + MOH \qquad (M = Li, Na)$$

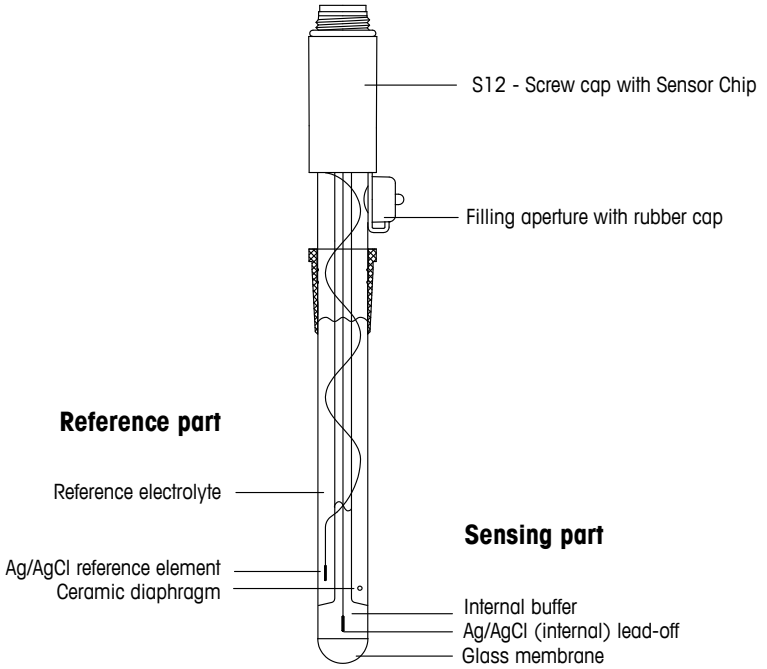
The alkali ions diffuse into the aqueous solution, leaving behind a virtually completely protonated Si-O skeleton. The internal glass membrane remains anhydrous.

At the phase boundary solution/hydrated layer, a thermodynamic equilibrium is established. This is possible only because the hydrogen ions in the hydrated layer are mobile. When the hydrogen ion concentration in both phases is different, hydrogen ion transport takes place. Every inflow and outflow of H^+ ions into or out of the glass membrane disturbs the electroneutrality of the hydrated layer. A potential is thus set up at the phase boundary that opposes further transport of H^+.

The number of hydrogen ions in the hydrated layer depends on the silicon oxide skeleton and is constant and independent of the nature of the analysis solution. Due to the movement of cations through the glass membrane, the electrical potential at the hydrated layer is transferred to the inner surface of the membrane where a hydrated layer with a phase boundary potential is also present. The total membrane potential results from the difference between the two phase boundary potentials.



In routine work a glass sensor with integrated reference electrode (so-called combination or combined sensor) is usually employed.



When the pH values in the two hydrated layers are identical (ideal case) and the pH value of the internal electrolyte (see figure) of the glass sensor remains constant, the membrane potential is given by

$$E_{membrane} = \text{Constant} + \frac{2.303 \cdot R \cdot T}{F} \cdot \log\left[H^+\right]$$

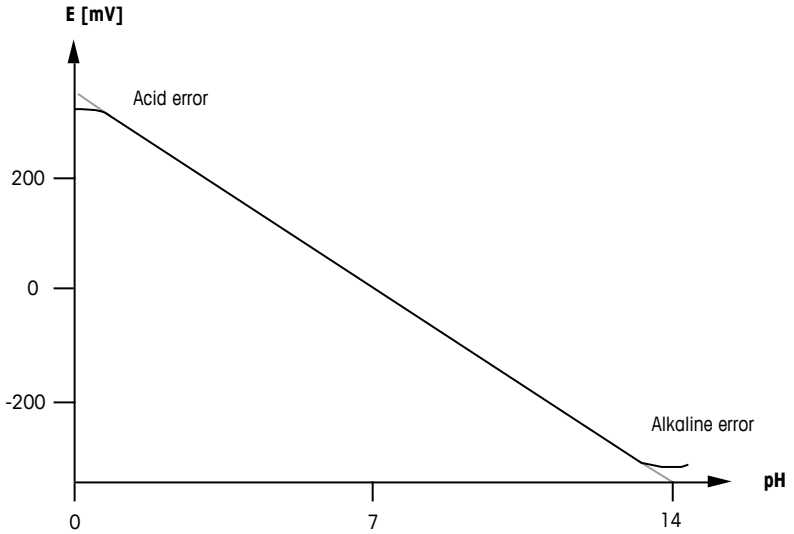
In practice, this equation is never satisfied exactly. Three factors contribute to the non-ideal behavior:

1. Asymmetry potential
The membrane potential should be zero when the glass membrane is in contact with identical solutions on both sides. Generally, however, a potential of a few mV is observed owing to the different history of the two sides of the membrane. A small asymmetry potential is not important since it is compensated in the calibration.

2. Alkaline error
In alkaline solutions the H^+ ions of the hydrated layer are partly replaced by alkali ions, especially sodium ions. The lower H^+ concentration in the hydrated layer thus leads to lower pH values.

The difference between the theoretical and experimental pH value is called the alkaline error. In the types of lithium-based glass used today, the pH deviations become significant above pH 13. The alkaline error increases with increasing pH value, increasing alkali concentration and increasing temperature.

3. Acid error
In highly acidic solutions (pH < 1) the glass sensor exhibits deviations from the ideal pH function. By uptake of acid molecules, the hydrogen ion activity of the hydrated layer is increased, thereby leading to positive pH shifts. The acid error is less disturbing than the alkaline error.



A further characteristic of the glass membrane is its high electrical resistance, which lies between 10 and 10 000 MΩ, depending on the composition of the glass, the temperature and the size of the membrane. This high resistance thus puts increased demands on the measuring system.

The internal electrolyte ensures a constant phase boundary potential at the inner surface of the glass membrane and a constant potential at the internal lead-off. Usually a silver wire coated with Ag/AgCl is used as internal lead-off, whose potential is determined by the chloride ion activity of the internal electrolyte.

4.1.5 Ion selective sensors

Ion selective sensors are electrochemical half cells in which a potential difference arises at the phase boundary electrode/solution that depends on the concentration (more correctly activity) of a specific ion in the solution.

Glass sensors are also ion selective sensors. The setup of an ion selective sensor assembly is similar to that of the pH sensor and comprises an ion selective membrane and a reference electrode of constant potential.

Instead of a pH scale, an ion scale is defined, for example a pNa or a pCl scale.

Many cations and anions, neutral gases such as NH_3, CO_2 and SO_2 and even organic substances such as amino acids can be quantitatively measured directly with ion selective sensors. Even ions or neutral substances that are not measurable directly can be determined indirectly if a chemical auxiliary reaction is applied where a substance that can be detected by a sensing electrode is released or bound.

In the ideal case the electrode assembly potential of an ion selective sensor is described by an expanded Nernst equation, the so-called Nicolsky equation:

$$E = E_0 \pm S \cdot \log\left(a_i + \sum_j K_{ij} \cdot a_j^{n_i/n_j}\right)$$

with:

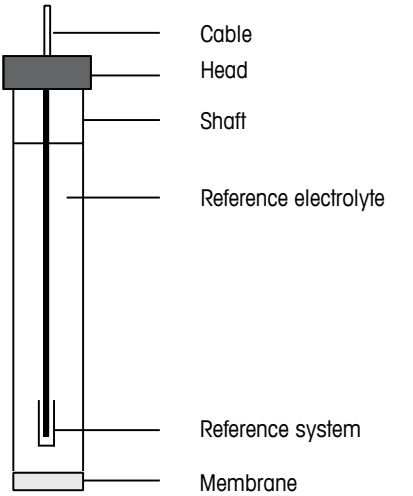
E: electrode assembly potential
E_0: electrode assembly potential at the reference point ($a_i = 1$, $a_j = 0$)
S: slope ($S = 2.301 \cdot R \cdot T/n_i \cdot F$). The sign is + for cations and – for anions.
a_i: analyte ion activity in solution
a_j: interfering ion activities in solution
K_{ij}: selectivity coefficients of interfering ions
n_i: charge number of analyte ion
n_j: charge numbers of interfering ions.

The selectivity coefficients of the interfering ions are a measure of the selectivity of the sensor. They should be as small as possible so that the interfering ions make no appreciable contribution to the ion selective potential change at the measuring cell. With a value of $K_{ij} = 1$ the contribution of the interfering ions to the potential change is exactly the same as that of the analyte ion (assuming same charge numbers). A completely selective sensor, i.e. one that responds only to one type of ion under all conditions does not exist.

The glass sensor has a selectivity coefficient towards sodium ions of $10^{-12} – 10^{-13}$, in other words good pH sensors are disturbed by sodium ions only at Na^+ concentrations greater than 0.1 mol/L and pH values above 12.

The astonishing selectivity of pH glass sensors is by no means attained by other ion selective sensors. Selectivity coefficients of 10^{-5}–10^{-6} are typical.

Practically all commercially available ion selective sensors are membrane sensors.



From the schematic setup of such a sensor it is apparent that at the membrane the potential is developed.

The shaft of the sensor is closed at its bottom end by the membrane and usually comprises an aqueous reference solution and an electrode of the 2nd kind. Various types of material are used for the membrane.

The membrane of the glass pH sensor are mostly made by glass blowing and fused onto the sensor shaft.

Solid-state membranes comprise crystal sections or homogeneous or heterogeneous pellets. The ionic conduction in the solid forms the basis of operation of these types of sensor.

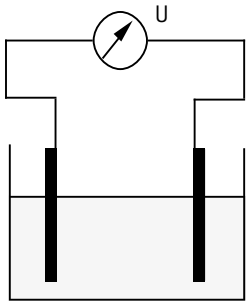
Liquid membranes comprise a porous carrier material containing a solution of an organic substance in an organic (immiscible with water) solvent. Today, gel membranes that contain the organic solution as a plasticizer in highly polymerized materials (e.g. PVC) are usually employed. The lifetime of such gel membranes is limited.

A double junction reference electrode is normally used as reference.

4.1.6 Measurement technique

All electrochemical measurements have the following in common: they are performed using an electrode assembly consisting of a sensing and a reference electrode.

Potentiometry
The direct measurement of the galvanic potential developed by an electrode assembly is called potentiometry, while the performance of a titration by use of this method is referred to as a potentiometric titration.



The potential U that develops should be measured, if at all possible, at zero current with a high impedance signal amplifier for the following reasons:

- The basis of potentiometry is the Nernst equation, derived for sensors in chemical and electrical equilibrium. An excessive current flow across the phase boundary surfaces concerned would disturb this equilibrium.
- A further reason for use of a high impedance measuring input results from the special construction of pH and ion selective electrodes. The measuring circuit includes the ion selective membrane, whose electrical resistance can easily be 100–1000 MΩ. If the experimental error due to the voltage divider effect is to be kept below 0.1%, the input impedance of the measuring instrument should be at least 1000 times greater. This can be seen from the following equation:

$$\text{Error in \%} = \frac{R_{\text{electrode assembly}}}{R_{\text{electrode assembly}} + R_{\text{input}}} \cdot 100$$

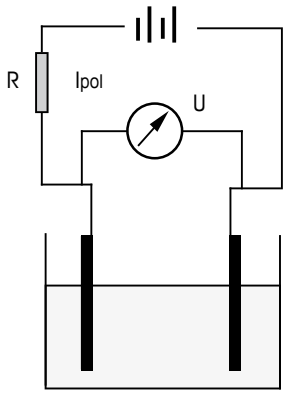
For very high resistance sensors, signal amplifiers with an input impedance of 10^{12} Ω are thus necessary.

Voltametry

This indication technique involves the measurement of the potential difference between two metal electrodes that are polarized by a small current.

As in the case of potentiometry, the voltametric titration curve is a potential-volume curve.

The following measuring equipment is needed:

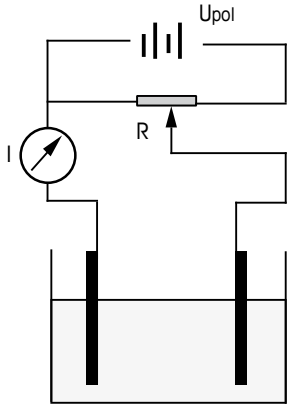


The stabilized power supply source provides the current. The resistance R connected in the circuit must be selected such that a current I_{pol} can be generated in the range 0.1 – 20 µA. The potential U that develops between the electrodes is measured exactly as in potentiometry.

One of the main applications of voltametric indication is the determination of water by the Karl Fischer method.

Amperometry

Amperometric indication like voltametric indication makes use of polarized electrodes but uses a constant potential instead of a constant current. The measured variable is here the current flowing through the electrodes and the titration solution. The amperometric titration curve is thus a current-volume curve. The following experimental setup is needed:



A constant potential U_{pol} is applied between the two electrodes with a voltage divider. The resulting current I is measured with a micro ampere meter.

4.2 Photometric indication

The basis of photometric indication is the decrease in intensity at a particular wavelength of a light beam passing through a solution. The transmission is the primary measured variable in photometry and is given by

$$T = \frac{I}{I_0} \qquad (\text{or } \%T = \frac{I}{I_0} \cdot 100)$$

T: Transmission
I_0: Incident light intensity
I: Transmitted light intensity

If all light is absorbed, then I = 0 and hence T = 0. If no light is absorbed, $I = I_0$ and T = 1 (or %T = 100%).

In photometry, work is frequently performed using absorption as the measured variable. The relation between transmission and absorption is described by the Bouguer-Beer-Lambert Law:

$$A = -\log T = \varepsilon \cdot c \cdot d$$

A: Absorption
ε: Extinction coefficient
c: Concentration of the absorbing substance
d: Path length of the light through the solution

From the above relation it can be seen that there is a linear relation between absorption A and concentration c.

In comparison with potentiometric sensors, photoelectric sensors have a number of advantages in titration:

- they are easier to use (no refilling of electrolyte solutions, no clogging of the junction)
- longer lifetime (they are virtually unbreakable)
- they can be used to perform all classical titrations to a color change (no change in traditional procedures and standards).

Photometric indication is possible for many analytical reactions:

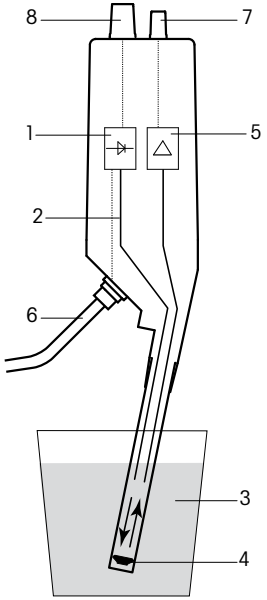
- Acid-base titrations (aqueous and nonaqueous)
- Complexometry
- Redox titrations
- Precipitation titrations
- Turbimetric titrations

In phototitration a wavelength should be selected which gives the greatest difference in transmission before and after the equivalence point. In the visible region such wavelengths are usually in the range 500 to 700 nm.

4.2.1 The METTLER TOLEDO phototrode

The METTLER TOLEDO DP5 Phototrode is a probe for photometric titration in the visible region. It provides a choice of 5 wavelengths comprising 520 nm, 555 nm, 590 nm, 620 nm and 660 nm.

The measurement principle is shown schematically in the following figure:



The photodiode (1) built into the probe emits modulated light that passes through the sample liquid (3) via light guide (2). The light reflected by the concave mirror (4) is converted by the detector (5) into an electrical signal that is amplified and led to the titrator via connection (6). The signal amplification can be adjusted by means of a control knob (7). Switching between the five different wavelengths is achieved by turning the dial (8).

4.3 Special indication methods

4.3.1 Conductometric indication

Conductometric indication [1], [2] [3] makes use of the ability of aqueous solutions to conduct an electric current. This conductivity is based on the dissociation of acids, bases and salts into electrically charged species (ions) in aqueous solution. In an electric field the anions migrate to the positively charged anode and the cations to the negatively charged cathode. Faraday's law states that per mole equivalent entity the same quantity of electricity, namely 96'485 coulombs, will be transported to the electrodes.

The conductivity of a dilute electrolyte solution depends on
- the ionic concentration
- the charge number of the ions
- the mobility of the ions in the solvent
- the polarity of the solvent
- the temperature (the conductivity increases by around 2.5% per degree Celsius).

The electrical conductivity is determined by measurement of the resistance. The measured resistance R depends on the separation l and the cross-sectional area q of the electrodes:

$$R = \rho \cdot \frac{l}{q}$$

The proportionality factor r is called the resistivity. The following relation holds between the conductivity and the resistivity:

$$\rho \cdot \chi = 1$$

The conductivity is thus obtained from the measured resistance R and the dimensions of the conductivity cell:

$$\chi = \frac{1}{R} \cdot \frac{l}{q} = G \cdot Z$$

The factor 1/R is also known as the conductance G. The conductance has dimensions µS or mS (S = Siemens). The quantity 1/q is referred to as the cell constant Z. The cell constant has dimensions of cm^{-1}. Typical values are between 0.1 and 10 cm^{-1}. It is always specified by manufacturers of conductivity cells and should be selected to match the concentration of the solution being titrated.

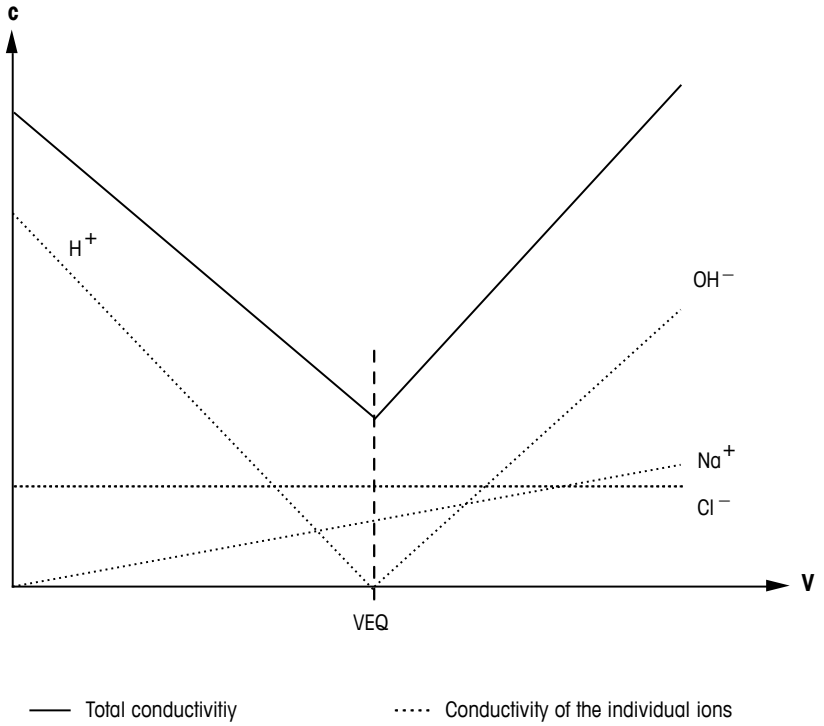
The conductivity should be specified in units of µS/cm or mS/cm, depending on its magnitude.

For the determination of the conductivity, alternating current must be used for the resistance measurement. If a direct current flows between the electrodes, electrolysis takes place and the contribution of the ohmic resistance, which is the sole variable of interest, becomes so small that its measurement is impossible.

The practical application of conductometric indication is limited to acid-base and precipitation titrations.

The characteristic profile of conductometric titrations is exemplified by the titration of hydrochloric acid with sodium hydroxide:

$H^+ + Cl^- + Na^+ + OH^- \rightarrow Na^+ + Cl^- + H_2O$



The measured conductivity at every point on the titration curve is composed of the sum of the conductivity of the individual ions. The titration diagram overleaf shows the contributions of the individual ions to the total conductivity (dilution not taken into account).

The titration curves are actually straight lines as long as each ionic species present reacts quantitatively or not at all. The typical curve character is due to the fact that one ionic species of the solution disappears (in our case H^+) only to be replaced by a new one from the titrant (here OH^-). When the equivalence point is exceeded, an increase in the conductivity is always observed in the absence of any further reaction.

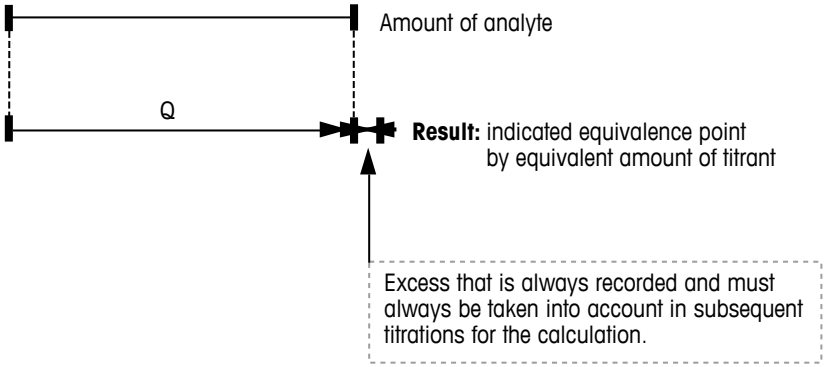
[1] F. Oehme, "Angewandte Konduktometrie", Hüthig Verlag, Heidelberg (1961)
[2] F. Oehme, "ABC der Konduktometrie", offprint Chemische Rundschau (1979)
[3] E. Pungor, "Oscillometry and Conductometry", Pergamon Press, Oxford (1965)

5 Titration types

Titrations can be classified in various ways. The classification by means of indication method and analytical reaction has been discussed in earlier sections. This section describes the classification of titrations according to the manner in which they are performed.

5.1 The direct titration

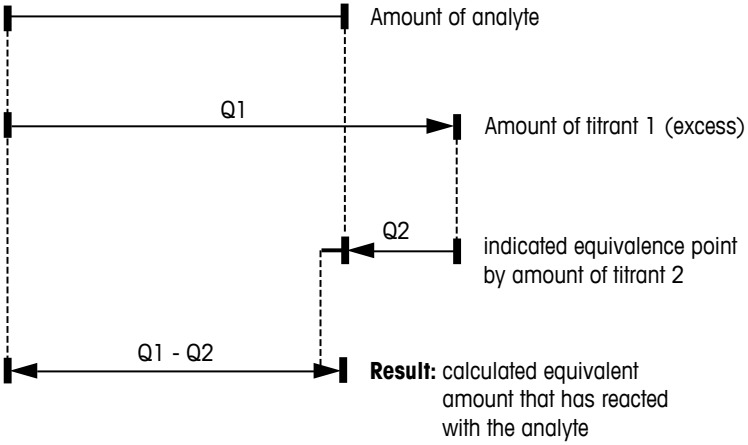
In the direct titration the titrant reacts directly with the analyte. The performance of a direct titration can be represented as follows:



Under the experimental conditions usual in the practical procedure, not every reaction fulfills the requirements described in section 1 for the titration reaction. Further, under certain circumstances indication of the equivalence point may also be poor. In such cases an indirect method is often employed to obtain the result.

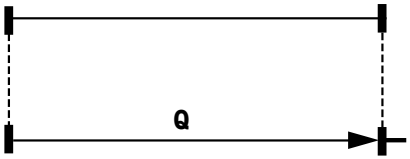
5.2 The back titration

In a back titration an excess of titrant is added to the sample. After a sufficiently long waiting time, this excess is then backtitrated with a second titrant. The difference between the added amount of the first and second titrant then gives the equivalent amount of the analyte. The back titration is used mainly in cases where the titration reaction of the direct titration is too slow or direct indication of the equivalence point is unsatisfactory.



5.3 The inverse titration

By initial addition of a metered volume of titrant followed by titration with the sample solution (= reverse of titration), the titration reaction may, under certain circumstances, be faster than in the direct titration. The classic example of an inverse titration is the determination of sugar using Fehling's solution.



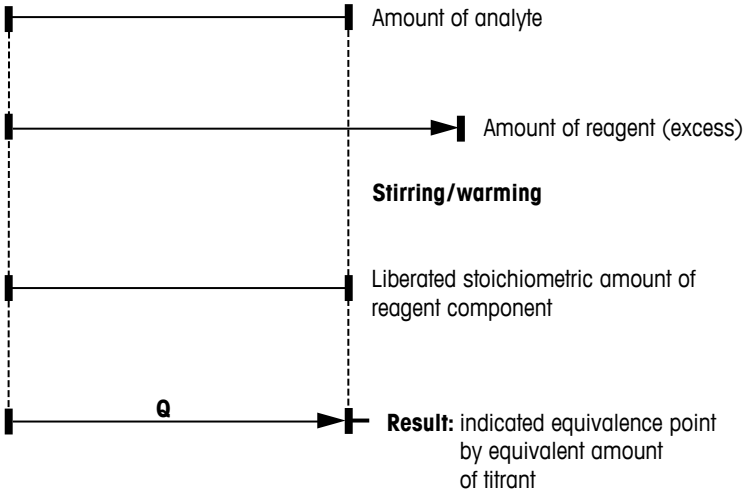
Amount of titrant added initially

Result: indicated equivalence point by equivalent amount of the analyte

5.4 The substitution titration

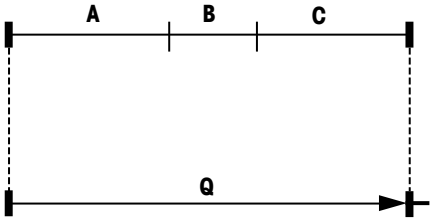
The action of the substitution titration is based on the addition of a reagent to the sample solution that reacts with the analyte. Here, a component of the added reagent is released in a stoichiometric amount and is then determined by direct titration.

Example:
Iodometric determination of copper (see section 3.2.5)



5.5 The collective titration

In a collective titration the sum of the components is determined as an equivalent amount. An example of a sum titration is the complexometric determination of water hardness (Ca + Mg) by titration with EDTA. Acid-base and redox titrations are also often performed as a collective titration.



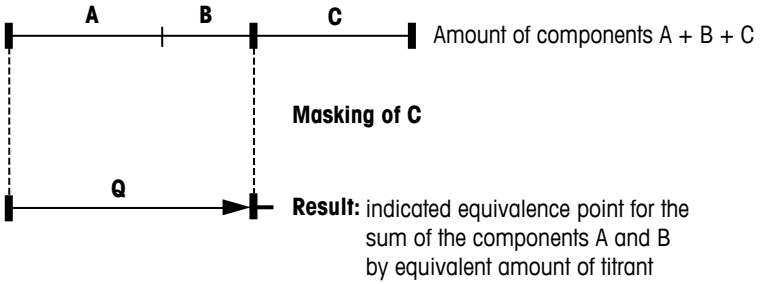
Amount of components A + B + C

Result: indicated equivalence point for thesum of the components A, B and by equivalent amount of titrant

5.6 The selective titration

By a suitable choice of experimental conditions – such as pH and masking agent – sum titrations can be performed completely or partly selectively with the use of suitable titrants.

Example:
Masking of iron with triethanolamine in the complexometric determination of calcium and magnesium with EDTA.



5.7 The sequential titration

In a sequential titration various components of a sample can be determined with just one titrant. Sequential titrations are selective when the equilibrium constants of the titration reactions of the individual components differ sufficiently. In a mixture comprising two or more components, the element that forms the most stable complex with the titrant is removed by titration first.

Acid-base titrations
Selective acid-base sequential titrations are possible when the pK value of the different acidic or basic components differ by at least two units. The choice of a nonaqueous solvent often allows an improved differentiation.

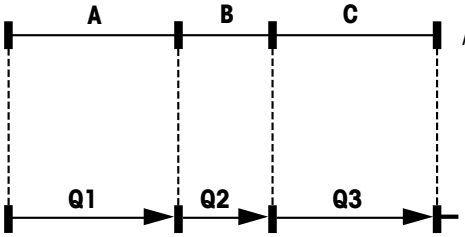
Complexometric titrations
In complexometric sequential titrations the following criteria must be met:

- the effective stability constants must show a difference of at least five logarithmic units
- the minimum value of the relevant stability constant (in logarithmic units) must be at least seven.

Redox titrations
Selective redox titrations are also possible. The potential difference between the equivalence points must be at least 300 mV.

The principle of the sequential titration is summarized in the following diagrams:

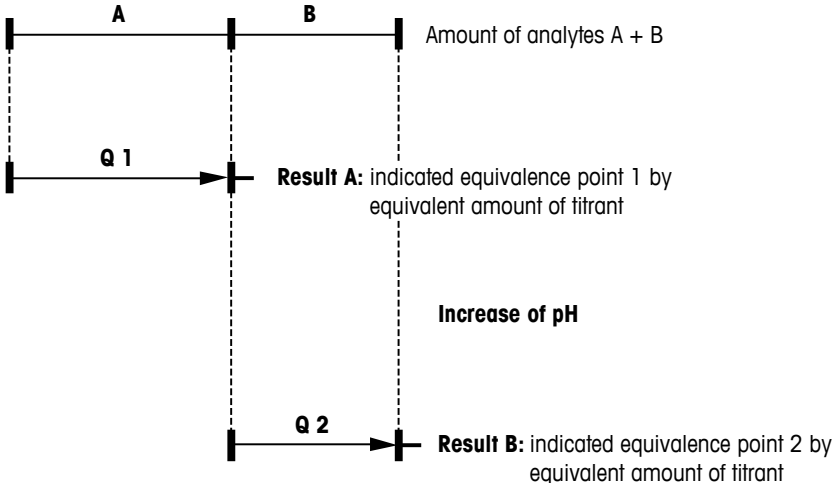
Simple sequential titration:



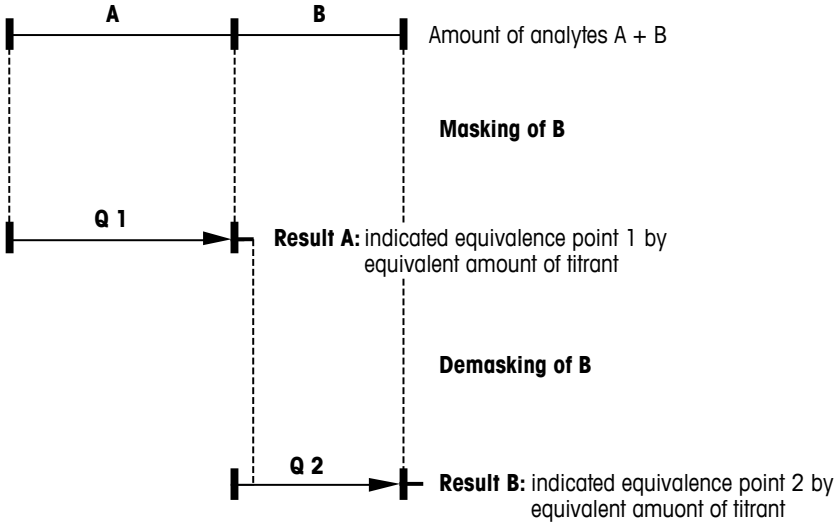
Amount of analytes A + B + C

Result A, B, C:
indicated equivalence points 1, 2 and 3 by equivalent amount of titrant in each case

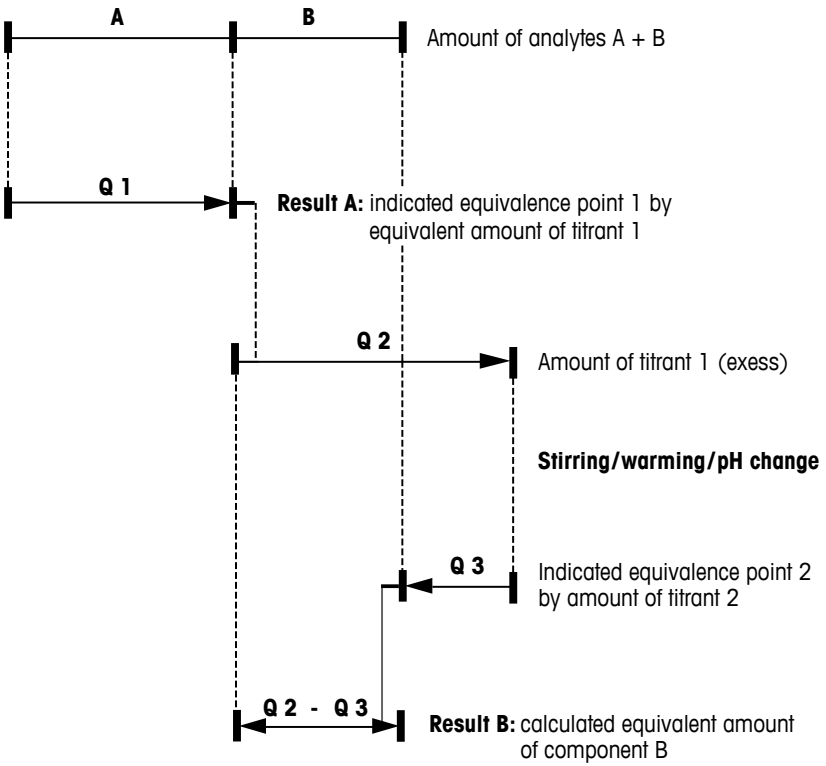
Sequential titration with change in the pH value:



Sequential titration with masking/demasking:



Sequential titration comprising a direct titration and a back titration:



6 Titration curves

Titration curves illustrate the qualitative progress of a titration. They allow a rapid assessment of the titration method. A distinction is made between logarithmic and linear titration curves.

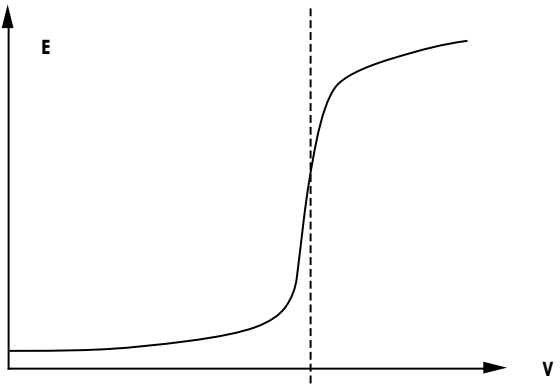
Logarithmic curves appear when the measured signal has a logarithmic dependence on the equilibrium concentration. Such curves are obtained with all indication methods that follow the Nernst equation. This includes all potentiometric titrations.

If there is a linear relation between the measurement signal and the concentration, the titration curves are said to be linear. The most important application is photometric titration. Further examples are titrations with conductometric, amperometric and thermometric indication.

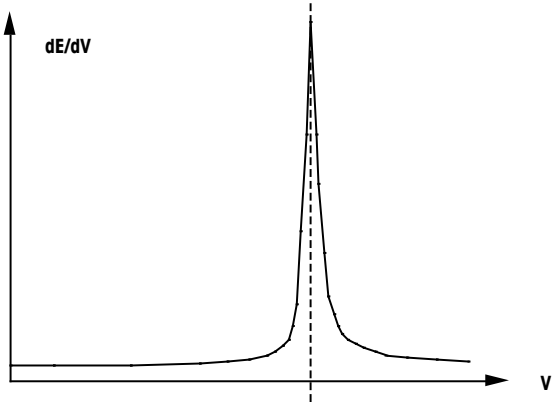
6.1 Measurement signal as a function of the titrant volume: E = f(V)

This representation can be used for the graphical determination of the equivalence point (see section 8).

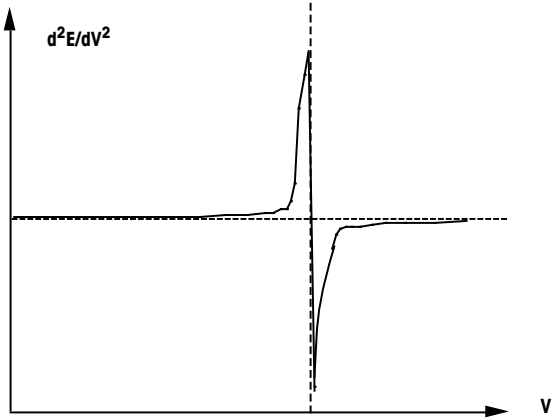
The equivalence point is approximately coincident with the inflection point of the titration curve.



If a plot of the first derivative of the curve is selected, the equivalence point is located in the vicinity of the maximum (with rising curve) or minimum (falling curve).



In the case of the second derivative, the equivalence point corresponds to the zero crossing.

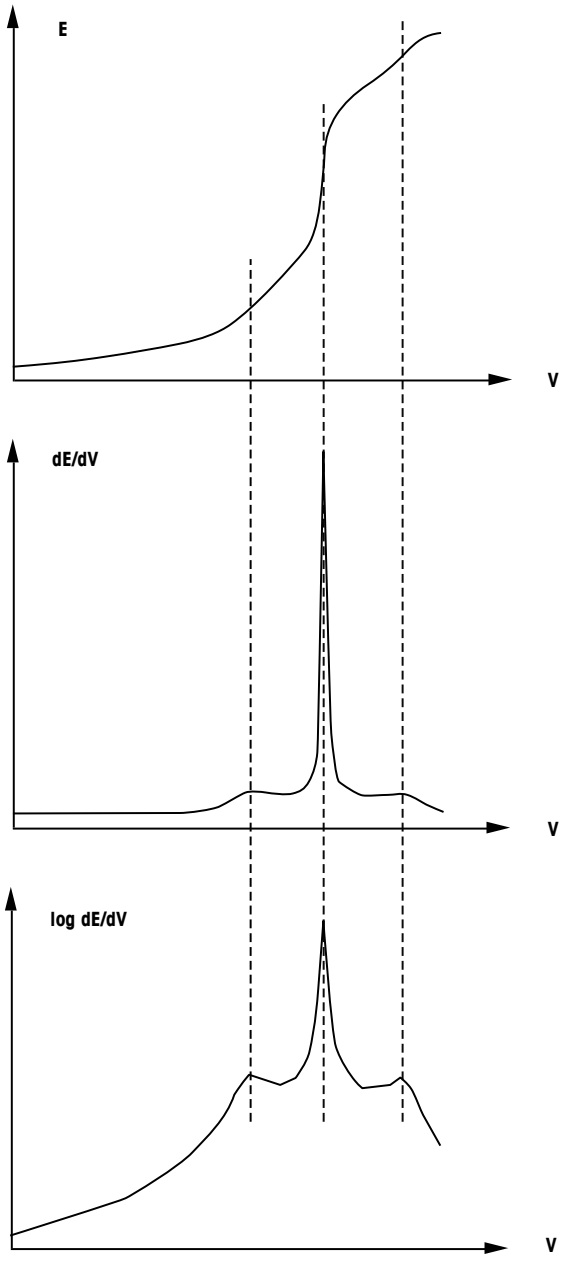


In titrations of mixtures the minimum/maximum values of the first derivative of the curve are in some cases not readily apparent when the titration curve comprises flat jumps in addition to a very steep jump. For such cases the following logarithmic representation of the first derivative is useful:

$$\log(dE/dV) = \mathrm{sign}(dE) \cdot \log(|dE/dV| + 1) = f(V)$$

$$\mathrm{sign}(x) = 1, \quad \text{if } x >= 0$$
$$\mathrm{sign}(x) = -1, \quad \text{if } x < 0$$

This representation gives greater prominence to small maxima over relatively large ones. This is illustrated in the following titration curve obtained y the acid/base titration of a mixture consisting of three acids.

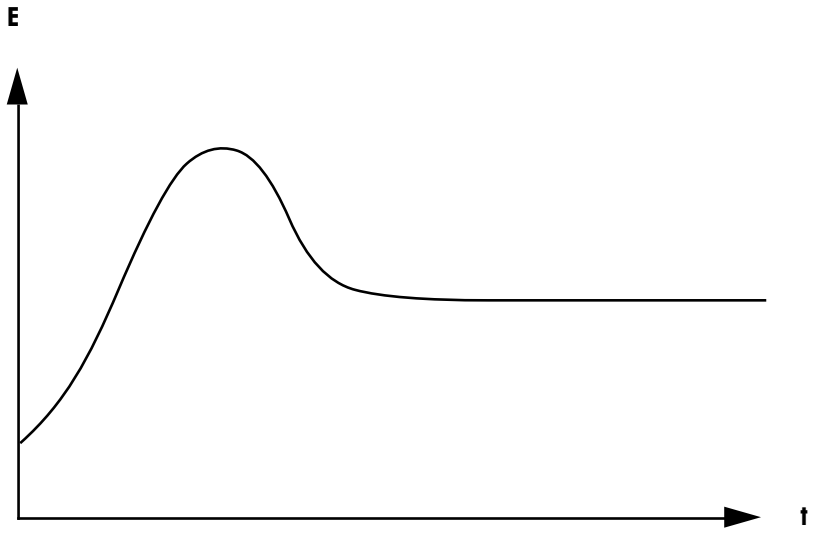

E
V
dE/dV
V
log dE/dV
V

6.2 Measurement signal as a function of time: E = f(t)

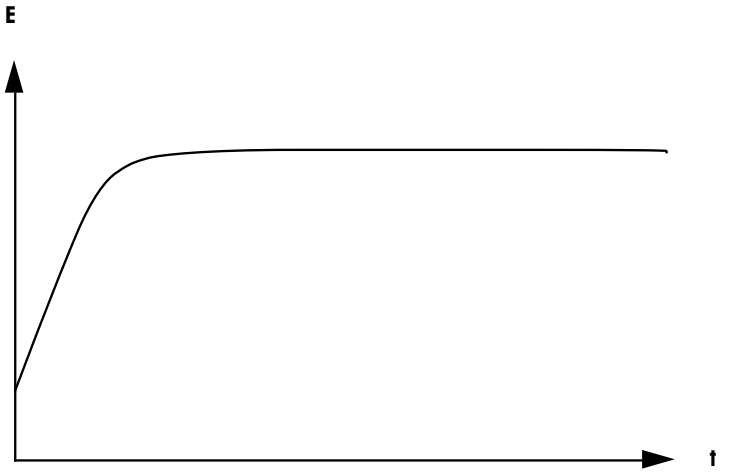
Representation of the signal as a function of time supports the development of new methods and the optimization of the equilibrium condition for the measured value acquisition. The time dependance of the measurement signal allows assessment of the response behavior of the sensor and the rate of the titration reaction.

The following figures show a few representative examples. The shape of the curve is influenced by the following parameters:

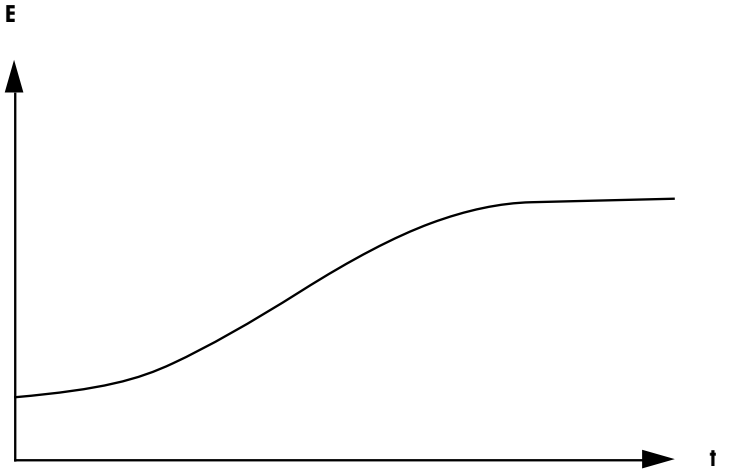
- response time of the sensor
- rate of the titration reaction
- stirrer speed



The example above shows the case of a slow reaction observed using a sensor with a fast response. The abrupt change in the signal at the start shows the immediate response of the sensor to addition of the titrant. The stabilization of the equilibrium signal is the result of the subsequent titration reaction.



The curve above reveals a typical example of a rapid reaction with a sensor having a rapid response. The sensor can follow the chemical reaction instantaneously, hence the exponential profile of the time signal.



A rapid reaction with the use of a slowly responding sensor is demonstrated by this example. It will appear often when work is performed with a dirty or poorly maintained sensor. The sensor does not respond until after a certain incubation time, but meanwhile the chemical reaction is already well advanced.

The concepts of "rapid" and "slow" are naturally relative in this context. They describe the rate of the titration reaction relative to the response behavior of the sensor.

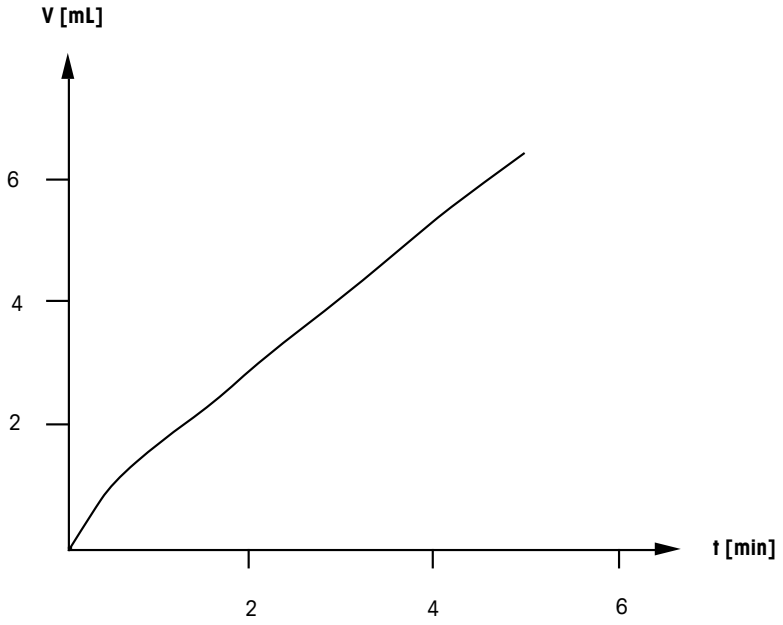
6.3 Titrant volume as a function of time: V = f(t)

This form of the titration curve – especially the first derivative dV/dt = f(t) – is an important representation of pH-stating reactions and Karl Fischer titrations.

- The amount of titrant added as a function of time describes the progress of the chemical reaction. The rate of titrant addition dV/dt is directly proportional to the reaction rate.

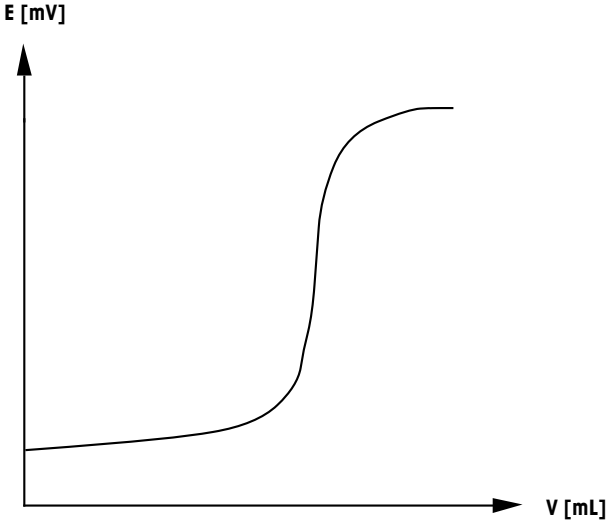
- In Karl Fischer titrations the representation of the rate of titrant addition – expressed in µg H_2O/min – as a function of time allows a simple assessment of the drift before and above all after the titration.

The following pH-stat titration curve obtained by the determination of the activity of pancreatic lipase 250 serves as a V/t curve example.



7 Control of the titration

A titration curve is represented by the measured signal E (unit: mV or a quantity derived from it such as pH) and the volume V of the added titrant (unit: mL). The signal describes the dependence of the progress of the titration reaction on the titrant addition.



In the Excellence and Compact titrators the titrant addition and measured value acquisition are intimately linked by a control system which ensures an accurate and repeatable titration result within the shortest possible time.

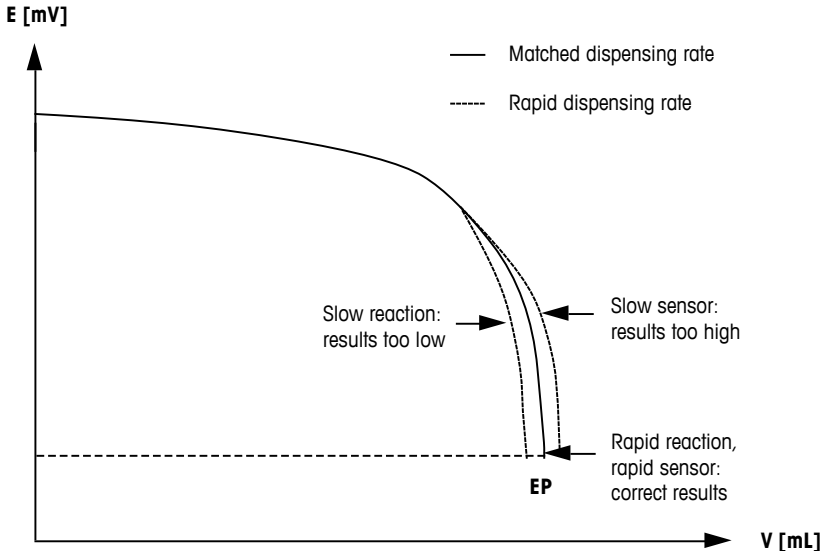
7.1 Titrant addition

The titrant can be added in two ways: continuously at a defined dispensing rate or incremental with individual volume. The volume steps are either defined in fixed increments or dynamically, dependent on the actual measured potential.

7.1.1 Continuous titrant addition: the end point (EP) titration

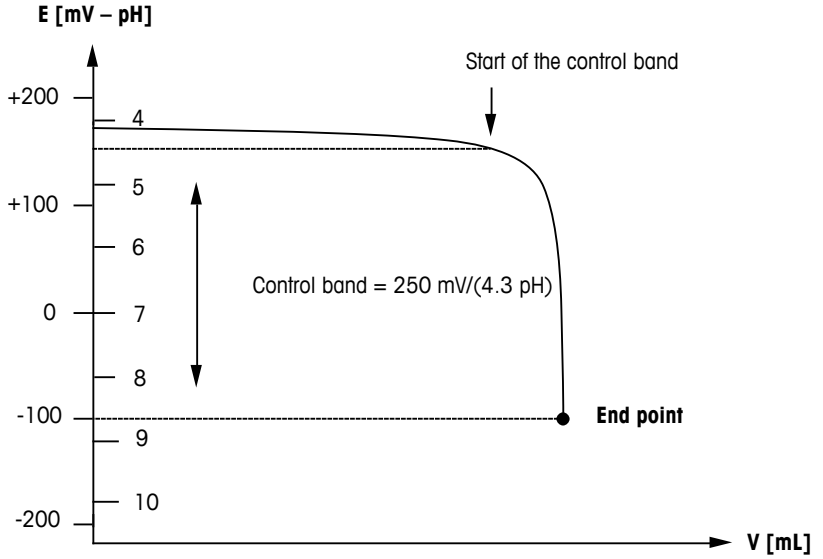
The continuous addition of titrant is the classic way to perform a titration with motorized piston burettes.

The dispensing rate must be matched to the rate of the titration reaction and the response time of the sensor. Particularly in cases where a large potential change occurs in the vicinity of the end point, diffusion phenomena appear at the junction and lead to a delay in stabilization of the potential. If the dispensing rate is too high, with a slow reaction the result is too low and with a slow response time of the sensor too high.



The Excellence titrators solve the problem with a variable dispensing rate. The titrant addition is controlled as a function of the measured signal change such that a distortion of the titration curve due to lag of the potential adjustment is avoided even in the transition interval.

The titrant addition is determined on the one hand by using the distance of the actual potential to the end point and on the other by using dE/dt. The closer the potential to the end point, the slower the titrant addition. Thus, if the control band width is increased, the control reacts more sensitively to potential changes within the control band, which leads to a stronger reduction of the dosing speed.

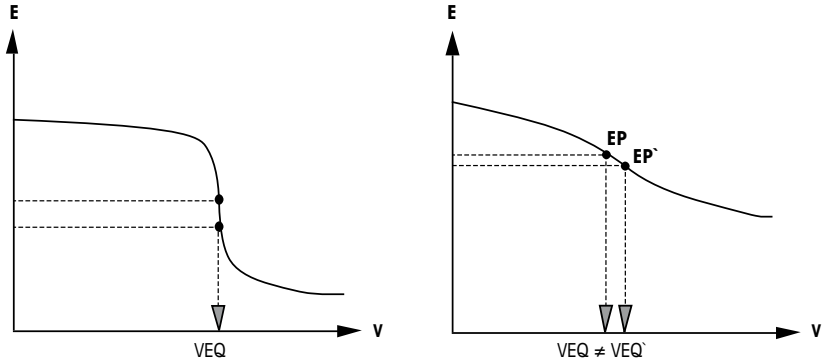


A large control range leads to an accurate but slow titration. A narrower range results in a rapid titration, but there is an inherent danger of overtitrating. With an S-shaped, steadily progressing titration curve only about the last 10% of the volume to be added should lie within the control range.

The time from the attainment of the end point up to the definitive termination of the titration is called the delay. If the signal of the sample solution deviates from the original end point signal during this time, additional increments are added until the end point is again reached. A large value of the delay (typical value: 10 s) should be selected with:

- large titration vessels
- inefficient stirring
- slow analytical reaction
- long response time of the sensor

The continuous titrant addition in end point titrations is suitable only for steep titration curves. With flat curves (see figure), wrong selection of the end point (EP′ instead of EP) or a drifting sensor leads to a false equivalence volume (VEQ′ instead of VEQ). For traditional reasons (old standards, procedures), however, even when the curves are flat, titration must sometimes be performed to the preset end point by means of continuous titrant addition. Before such determinations the appropriate sensor must always be calibrated to allow detection of the exact end point.



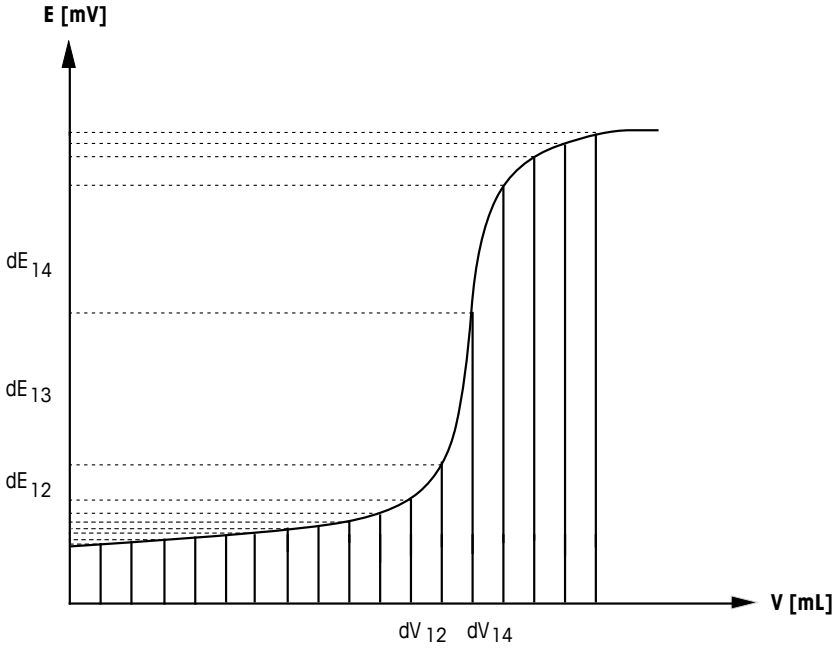
7.1.2 Incremental or dynamic titrant addition

Titrant addition by fixed increments

The accuracy of the result of a titration depends principally on the quality of the experimental data available for the evaluation. This led to the development of the incremental titrant addition.

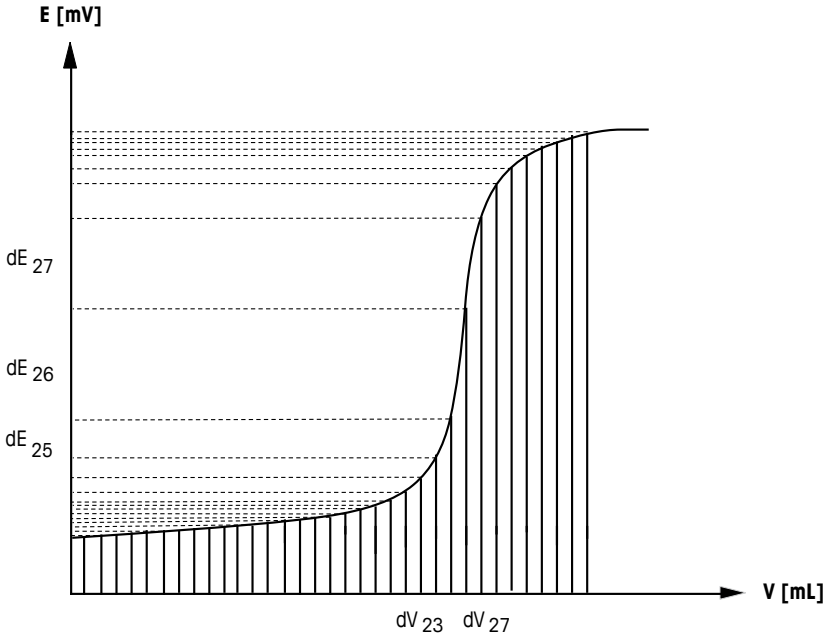
The titrant is added in single volume steps. After dispensing, the change in the signal is recorded accurately. Thus, for each dispensing step, the measurement method yields a measured value, a data point, of the titration curve.

If constant, relatively large volume steps are added, with steep titration curves there are very few data points in the vicinity of the equivalence point (see figure below).



dV = constant

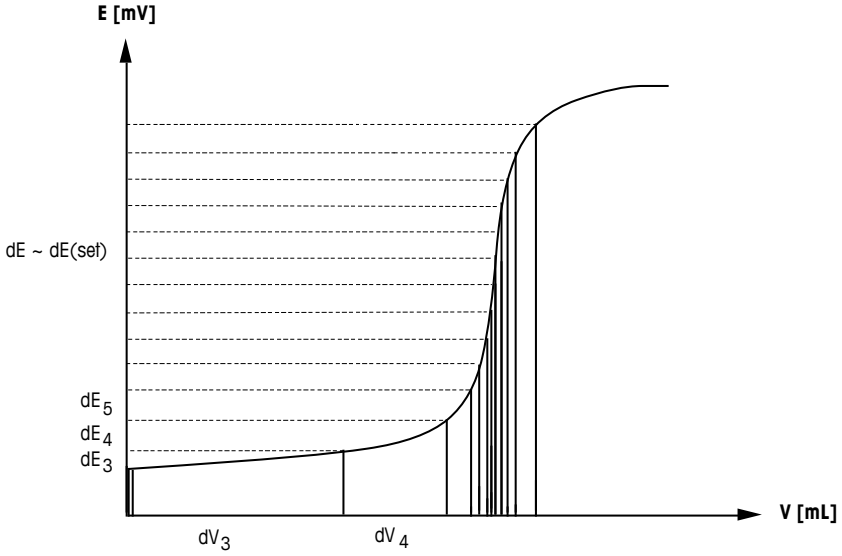
With smaller volume increments, more data points are obtained, but the titration takes longer and in the steep part of the curve, important for the calculation, the increase in the number of data points is relatively small.



dV = constant

Dynamic titrant addition

An obvious move is thus to size the titrant addition so that the signal change always remains about the same. This type of titrant addition is known as dynamic titrant addition and the corresponding titration control as dynamic titration. In the flat, unimportant part of the titration curve, large volume increments are dispensed and in the steep part of the curve, important for the calculation, smaller increments.



Dependent on the dE(set value) the volume increment dV is calculated. In addition, a lower limit dV(min) and an upper limit dV(max) are defined for the calculated dV. The choice of dE(set value), dV(min) and dV(max) depends on the shape of the titration curve. The recommended value of dE(set value) is determined by the height of the jump. For a jump height of e.g. 250 mV, a typical value of dE(set value) would be 12 mV. In very steep titration curves the calcuation of dV could in certain cases result in very small volume values. This can be prevented by a reasonable selection of dV(min), e.g. 0.005 mL. In the flat part of a titration curve, the calculation of dV can lead to unreasonably large volume increments. Limitation of dV(max) to e.g. 0.5 mL can prevent this.

Titration curve

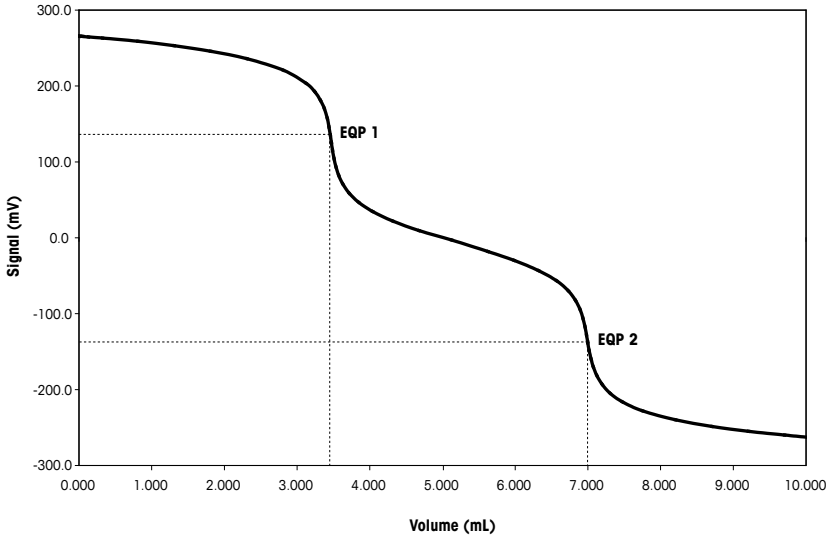


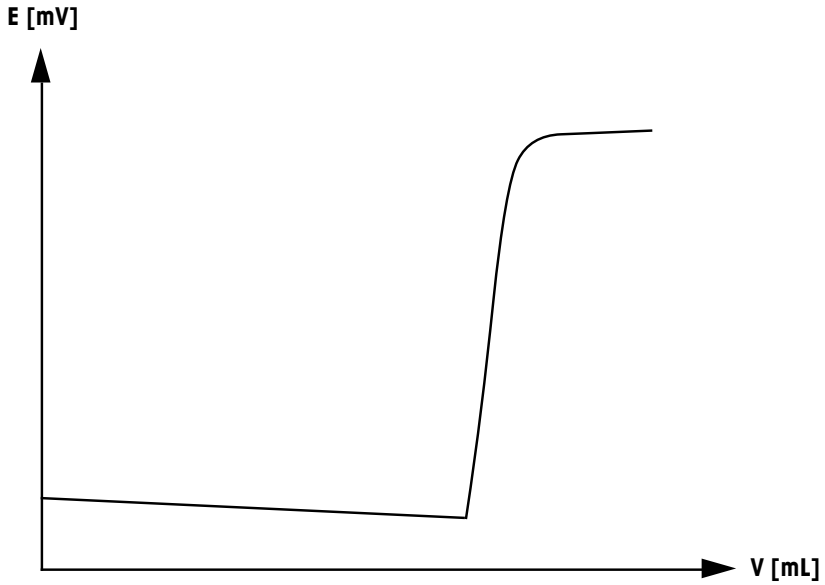
Table of measured values

	Volume mL	Increment mL	Signal mV	Change mV	1st deriv. mV/mL	Time s
	0.0000	NaN	265.7	NaN	NaN	0
	0.0050	0.0050	265.6	-0.1	NaN	3
	0.0100	0.0050	265.6	0.0	NaN	6
	0.0225	0.0125	265.5	-0.1	NaN	9
	0.0535	0.0310	265.3	-0.2	NaN	12
	0.1310	0.0775	264.7	-0.6	-7.6	15
	0.3250	0.1940	263.2	-1.5	-8.4	18
	0.8100	0.4850	258.9	-4.3	-8.4	22
	1.3100	0.5000	253.1	-5.8	-9.5	26
	1.8100	1 0.5000	245.8	-7.3	-14.6	30
	2.3100	0.5000	236.0	-9.8	-28.5	34
	2.8100	0.5000	220.8	-15.2	-59.6	37
	3.1220	0.3120	203.7	-17.1	-104.2	40
	3.2430	0.1210	192.6	-11.1	-150.4	43
	3.3210	0.0780	181.2	-11.4	-212.7	46
	3.3730	0.0520	170.6	-10.6	-303.4	49
	3.4150	0.0420	157.3	-13.3	-414.1	54
	3.4390	0.0240	146.0	-11.3	-447.9	58
	3.4565	0.0175	135.5	-10.5	-491.1	64
EQP1	3.4703	2 NaN	126.1	NaN	-548.3	NaN
	3.4720	0.0155	125.0	-10.5	-548.3	4 70
	3.4875	0.0155	115.2	-9.8	-474.0	77
	3.5080	0.0205	104.3	-10.9	-440.7	84
	3.5345	0.0265	93.4	-10.9	-388.3	92
	3.5725	0.0380	82.3	-11.1	-299.0	98
	3.6305	0.0580	70.7	-11.6	-213.9	104
	3.7170	0.0865	58.8	-11.9	-150.0	109
	3.8465	0.1295	46.9	-11.9	-104.2	114
	4.0400	0.1935	34.7	3 -12.2	-71.1	118
	4.3160	0.2760	22.1	-12.6	-47.7	121
	4.6785	0.3625	9.4	-12.7	-32.6	124
	5.1255	0.4470	-2.9	-12.3	-25.6	128
	5.6255	1 0.5000	-18.0	-15.1	-29.0	131
	5.9850	0.3595	-29.8	-11.8	-39.9	134
	6.3215	0.3365	-43.2	-13.4	-61.3	138
	6.5710	0.2495	-56.8	-13.6	-93.8	142
	6.7310	0.1600	-69.9	-13.1	-140.0	147
	6.8300	0.0990	-82.4	-12.5	-203.0	153
	6.8915	0.0615	-94.2	-11.8	-281.7	160
	6.9320	0.0405	-106.0	-11.8	-356.7	168
	6.9595	0.0275	-117.1	-11.1	-381.1	176
	6.9805	0.0210	-128.1	-11.0	-425.6	184
EQP2	6.9962	2 NaN	-137.1	NaN	-461.9	NaN
	6.9985	0.0180	-138.4	-10.3	-461.9	191
	7.0175	0.0190	-148.4	-10.0	-395.0	198
	7.0425	0.0250	-159.5	-11.1	-369.0	204
	7.0740	0.0315	-170.0	-10.5	-321.3	208
	7.1225	0.0485	-181.4	-11.4	-238.6	212
	7.1945	0.0720	-193.0	-11.6	-166.6	216
	7.3050	0.1105	-204.8	-11.8	-115.7	219
	7.4730	0.1680	-216.2	-11.4	-78.0	222
	7.7510	0.2780	-227.9	-11.7	-47.9	225
	8.2065	0.4555	-239.8	-11.9	NaN	228
	8.7065	0.5000	-248.6	-8.8	NaN	231
	9.2065	1 0.5000	-255.0	-6.4	NaN	234
	9.7065	0.5000	-259.9	-4.9	NaN	237
	10.0000	0.2935	-262.6	-2.7	NaN	240

1. The added volume increment never exceeds the value of dV(max) (here 0.5 mL) in the flat part of the curve.
2. The added volume increment is never less than the value of dV(min) (here 0.005 mL) in the steep part of the curve.
3. The signal change is always around i.e. close to the dE(set value) in the middle part of the curve thanks to the dynamic control.
4. The time between two increments varies from 3 s (t(min)) to 30 s (t(max)), depending on the parameters of the equilibrium-controlled measured value acquisition.

'NaN' is the abbreviation for 'not a number'. This indicates that at the specific point there are neither actual or derived measured values nor calculated values (1st derivative) available. This is due to the EQP evaluation/recognition algorithm which is explained in chapter 8.2.1.

If the titration curve exhibits a sharp kink before the equivalence point, a very small value of dV(max) must be selected to ensure that the jump is not missed. The value of dV(max) then approaches the value of dV(min). In such cases it is appropriate to perform the titration with small, constant volume increments.



7.1.3 Predispensing

Predispensing can shorten the time needed for titration considerably. There is no sense to acquire accurate data points during the initial stage of the titration if such data are not needed for the evaluation. The following predispensing modes are possible:

1. Predispensing to a fixed volume
2. Predispensing to a volume that is calculated by multiplying the factor with the sample size.
3. Predispensing to a certain potential value

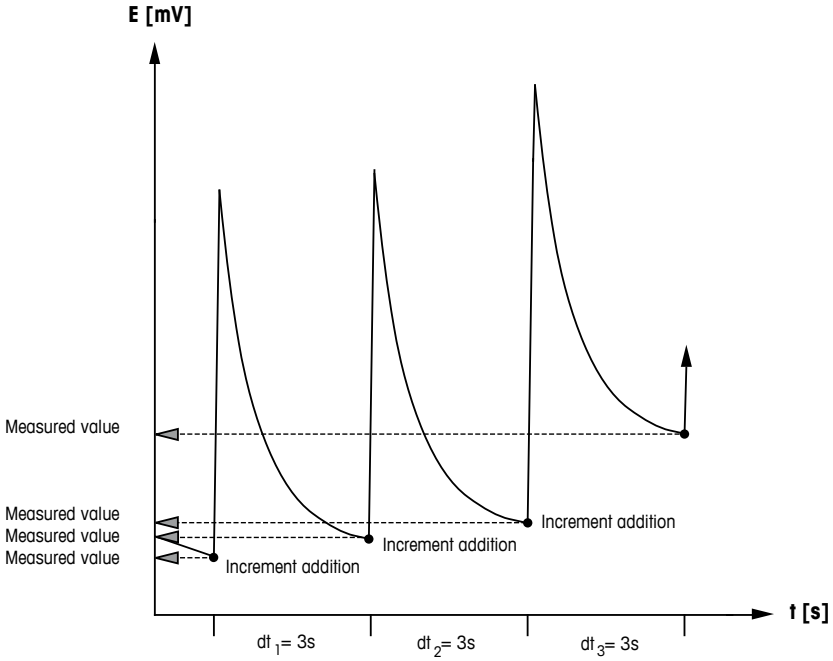
The first two modes need no signal measurement. The third mode requires control of the titrant addition with simultaneous measurement of the signal.

7.2 Measured value acquisition in EQP titration

After every addition of a volume increment a measured value must be acquired. This can occur in two ways:

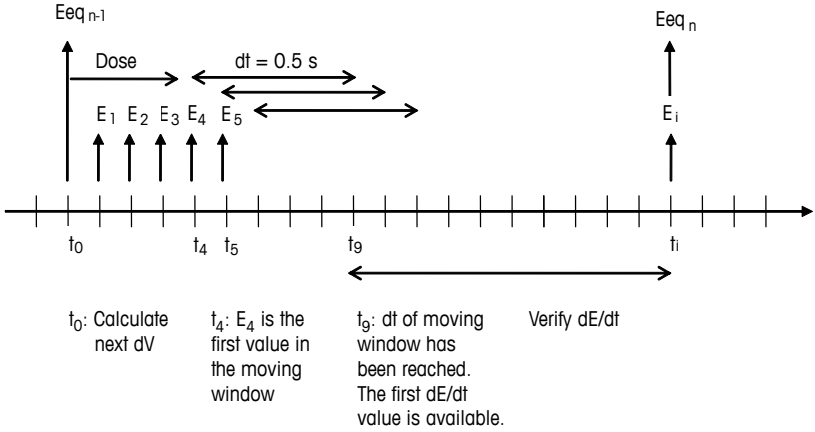
- Fixed time
- Equilibrium controlled.

In the Fixed time measured value acquisition the measured value is acquired following every increment addition after a certain waiting time dt.



In the equilibrium-controlled measured value acquisition the measured value after every increment addition is only acquired when an equilibrium has been established in the solution and the signal is stable.

The following diagram reveals the time course of the measured value acquisition during a titration.



Every 100 ms (t_i-t_{i-1}), a measured value E_i is sent to the controller. The time recording starts at t_0, i.e. after start of the dosing of the volume increment of the titrant. Measured values E_i are not considered during this time (E_1 - E_3). After accomplishment of the dosing step at t_4, a time window of the length dt (here 0.5 s) is filled with measured values E_i starting with E_4 every 100 ms. After dt has been elapsed at t_9, dE/dt (actual) is calculated by linear regression and verified against dE/dt (set) defined in the control parameters of the method. The measured value Eeq is only then recorded if the following conditions are met:

a. If the actual time t_i is greater than t_{min} and
b. If dE/dt (actual) is less than dE/dt (set)
c. If t(max) has been reached

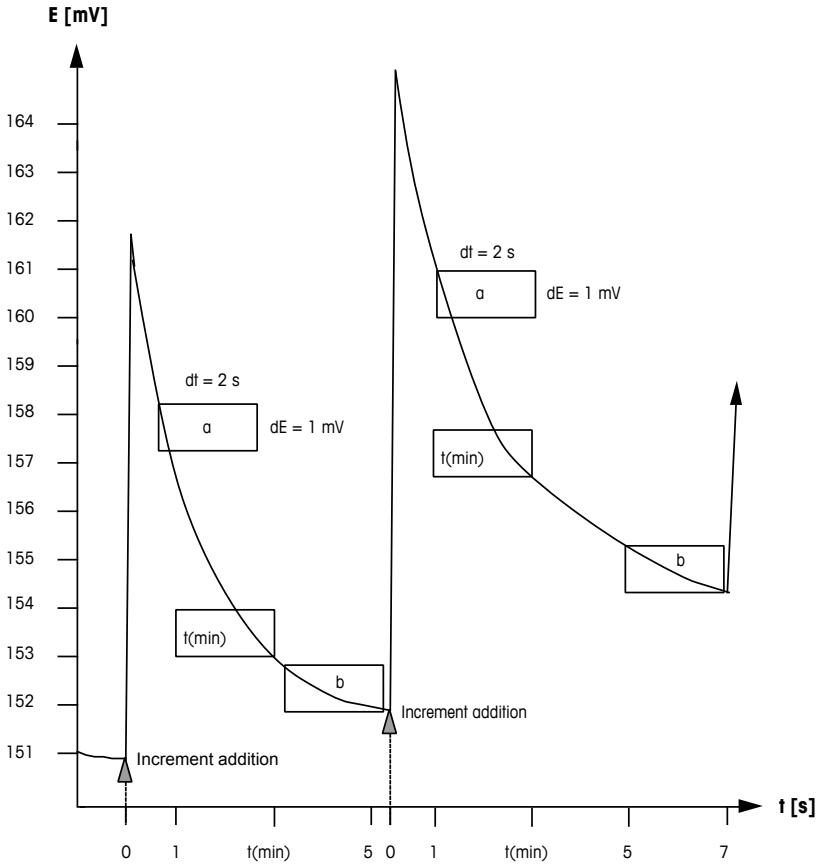
At t(max) the measured value is acquired at all events even if the equilibrium condition has not been met.

The measured values are acquired in this mode at different times. If the signal change after increment addition is small or the equilibrium is established rapidly, the measured value is acquired immediately after t(min) is exceeded. If the signal change is large or the equilibrium is established slowly, the waiting time is correspondingly longer. This ensures optimum matching of the measured value acquisition to the chemical reaction and to the response behavior of the sensor during the entire titration.

The following figure illustrates the measured value acquisition during titrant addition dependent on the set parameters as there are:
dE = 1 mV
dt = 2 s
t(min) = 3 s
t(max) = 30 s

For definition of the optimum equilibrium condition, observation of the signal as a function of time is necessary.

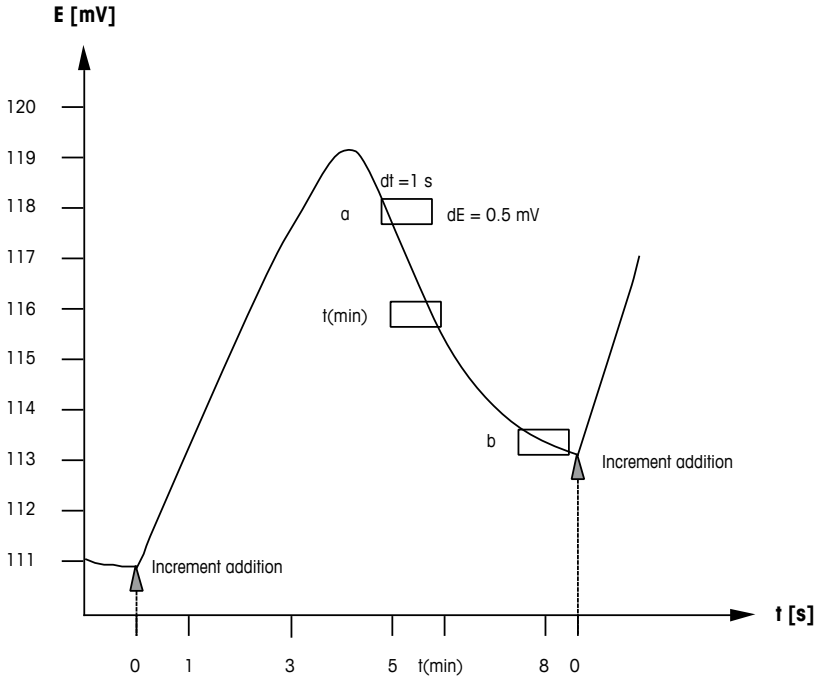
Note:
If the measured signal is very unstable (e.g. owing to noise, large fluctuations in the response behavior of the sensor, etc.) or if it drifts, preference should be given to the timed-increment measured value acquisition.



a: The defined equilibrium condition has not yet been met.
t(min): The equilibrium condition has not been met after 3 s.

b: The equilibrium condition has been met for the first time after 5.4 or 6.9 s.

If sensors have a slow response a slow response or if the analytical reaction is slow, a sufficiently large value of time t must be selected. With a large minimum time t(min), premature acquisition of the measured value in the case of an oscillating signal profile can be avoided (see section 6.2).



a: The defined equilibrium condition has not yet been met.
t(min): The equilibrium condition has not been met after 6 s.

b: The equilibrium condition has been met for the first time after 8.5 s.

7.3 Termination of the EQP titration

Termination of the EQP titration at the proper time can be triggered by selection of various parameters:

- Termination at maximum volume
- Termination after a certain number of recognized equivalence points
- Termination on attainment of a certain potential value. An additional parameter "Termination tendency" defines at which tendency the titration shall be terminated
- Termination when the slope of the titration curve falls below a specified value

The termination at maximum volume is an additional emergency measure to avoid, for example, overflow of the titration beaker if a malfunction occurs. Several termination criteria can be active at the same time; the one that is satisfied first causes termination of the unless the combination of the selected termination criteria is selected. In this case the titration is terminated as soon as all the selected criteria have been fulfilled.

7.4 Termination of the EP titration

Termination of the EP titration can be triggered by the following parameters:

- Termination at maximum volume
- Termination after passing of a delay time after the end point was reached
- Maximum time

8 The determination of the equivalence point

The end of a titration is reached when an amount of titrant equivalent to the substance being analyzed (the analyte) has been added. From the volume of the titrimetric solution required to reach this point – the equivalence point – and its known concentration, the amount of analyte can be calculated if the stoichiometry of the reaction is known. The correctness (precision and accuracy, see Section 10) of the result depends to a large degree on the method chosen to detect the equivalence point. The methods for recognition and exact calculation of the equivalence point are treated in this section.

8.1 The position of the equivalence point

In the immediate vicinity of the equivalence point, those titration curves that can be evaluated in practice exhibit a change in either the slope (so-called linear or segmented titration curves, e.g. titrations with amperometric, conductometric and photometric indication) or the direction of curvature (so-called logarithmic or S-shaped titration curves, e.g. titrations with potentiometric and voltametric indication). These break or inflection points are influenced by the equilibrium constants of the titration reaction, the initial concentrations, the change in volume due to the amount of added titrant and other factors. Each form of a titration curve in theory requires a specific type of equivalence point determination if optimum accuracy of the analysis result is to be achieved.

The end point of a potentiometric titration is often taken as the inflection point of the titration curve. The example of the titration of a strong acid with a strong base will be used here to demonstrate that determination of the inflection point provides a serviceable approximation of the equivalence point.

To represent the titration curve, the dependence of the H_3O^+ concentration or the pH value on the amount of added titrant is needed. The following table shows the H_3O^+ concentration or pH value and the extent of titration as a function of the amount of sodium hydroxide of concentration 0.1 mol/L added to a solution of 50 mL hydrochloric acid of concentration 0.01 mol/L.

mL NaOH	Extent of titration	c(H+) a*	pH a*	ΔpH a*	c(H+) b*	pH b*	ΔpH b*
0.0	0	10^{-2}	2		$1.0 \cdot 10^{-2}$	2.0	
4.5	0.9	10^{-3}	3	1	$0.917 \cdot 10^{-3}$	3.037	1.037
4.95	0.99	10^{-4}	4	1	$0.910 \cdot 10^{-4}$	4.041	1.004
4.995	0.999	10^{-5}	5	1	$0.909 \cdot 10^{-5}$	5.041	1.000
4.9995	0.9999	10^{-6}	6	1	$0.929 \cdot 10^{-6}$	6.036	0.995
5.0	1	10^{-7}	7	1	$1.0 \cdot 10^{-7}$	7	0.964
5.0005	1.0001	10^{-8}	8	1	$1.087 \cdot 10^{-8}$	7.964	0.964
5.005	1.001	10^{-9}	9	1	$1.100 \cdot 10^{-9}$	8.959	0.995
5.05	1.01	10^{-10}	10	1	$1.101 \cdot 10^{-10}$	9.958	0.999
5.5	1.1	10^{-11}	11	1	$1.110 \cdot 10^{-11}$	10.955	0.997
10.0	2	10^{-12}	12	1	$1.200 \cdot 10^{-12}$	11.921	0.966

a*: Dilution through addition of NaOH titrant is ignored
b*: Dilution through addition of NaOH titrant is considered

From the pH values and their differences shown in the table, it follows that even when the dilution due to addition of the titrant is taken into account, the titration curve is practically symmetrical in the region of the equivalence point (extent of titration ~1) and exhibits an inflection point that coincides with the equivalence point within experimental error.

In titrations of strong acids with strong bases, the difference in volume between the equivalence point and inflection point depends only on the dilution and is negligibly small. If the titration curve is highly asymmetric, the error in the determination of the inflection point can be so large that another type of calculation of the equivalence point becomes necessary (e.g. with heterovalent redox or precipitation titrations).

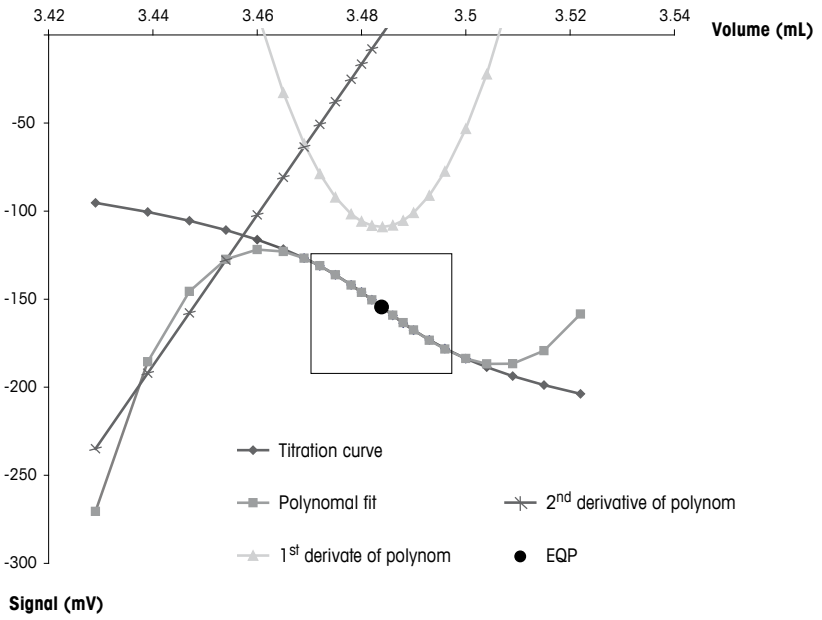
8.2 The practical detection and evaluation of the equivalence point

The following chapters describe the procedures for detection and evaluation of the equivalence points of symmetrical, asymmetrical and segmented shaped titration curves which are implemented in the Excellence and Compact titrators. The GRAN evaluation in chapter 8.2.6 shall serve as an example for a mathematical procedure. It is not available in the Excellence or Compact titrators.

8.2.1 Symmetrical S-shaped titration curves

Several measured values of a titration curve form a window. A suitable mathematical function is calculated that fits the section of the titration curve within that window. This function is now analyzed for inflection points. After each new measured value the next function within a window is calculated and analyzed for an inflection point and so forth.

The mathematical function that is used to fit an S-shaped section of a titration curve is a 3rd grade (cubic) polynom. Such polynoms are favorable since they can be easily calculated by a general linear least square algorithm, can be easily derived and fit parts of an S-shaped titration curve, which is a basic requirement for an inflection point detection.



The graphic illustrates the principle of a titration curve fitting by a 3rd grade (cubic) polynom. It shows a critical part of a titration curve, where an inflection point is suspected at approximately 3.48 mL. The black window circumscribes a defined number of points, which are fitted by a cubic polynom (line dotted with quadratic marks). Within the window the curve fit is excellent, whereas outside that window the divergence between the mathematical fit and the actual titration curve is huge. This divergence is due to the fact that a cubic polynom does not correspond to a chemical model, i.e. the complete titration curve cannot be simulated. The second derivative of the polynomal fit (line dotted with crosses) of the window section of the titration curve shows a zero point at 3.484 mL, which corresponds to an inflection point, i.e. the EQP. The titrant volume at the inflection point is a result of a mathematical calculation, i.e. no corresponding actual measured signal values are available for this point, which are therefore indicated in the table of measured values as NaN, the abbreviation of 'not a number'.

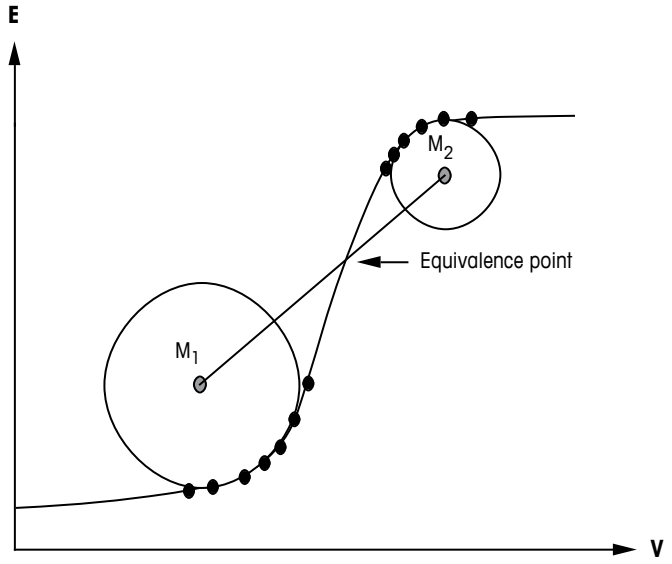
In the steep part of a titration curve, it is possible to detect inflection points in several adjacent windows. In order to select only one, i.e. the one with the best fit, the standard deviation of the measured values with respect to the curve fit is used as a criterion, i.e. only the best curve fit revealing the lowest standard deviation is used for selection of the inflection point.

8.2.2 Asymmetric titration curves

The Tubbs procedure

Tubbs [1] has described a graphical procedure for the evaluation of asymmetric, analog recorded titration curves. It has well proved its worth in routine analyses, as titration curves often do not exhibit the theoretical profile predicted by a mathematical model (e.g. precipitation and redox titrations).

The empirical method is based on the following idea:



A circle of curvature of minimum radius can be drawn in both branches of the titration curve. The ratio of the two radii is determined by the asymmetry of the curve. The intersection point of the straight line drawn between the midpoints M_1 and M_2 of the circles with the titration curve represents the equivalence point. Theoretical calculations show that the true equivalence point with asymmetric titration curves always lies between the inflection point and that branch of the titration curve with the greater curvature (the smaller circle of curvature). The result of the Tubbs evaluation approximates this true equivalence point very closely when the titration curve profile is regular and allows calculation of the circles of curvature of the two branches.

A mathematical variation of the Tubbs procedure for digitally recorded titration curve has been described by Ebel [2].

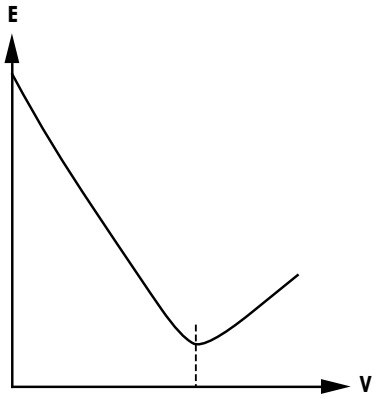
This involves approximation of those parts of the titration curve that lie in the region of the greatest curvature by a hyperbola. For each approximated hyperbola the vertex is determined. This point on the hyperbola lies at the position of greatest curvature. The midpoints of the assigned smallest circles of curvature are the foci of the two hyperbolae. As in the graphical version, the intersection point of the straight line joining the two foci with the titration curve gives the equivalence point.

The evaluation requires at least six measured points in the region of greatest curvature, both before and after the inflection point of the titration curve (see illustration).

8.2.3 Segmented titration curves

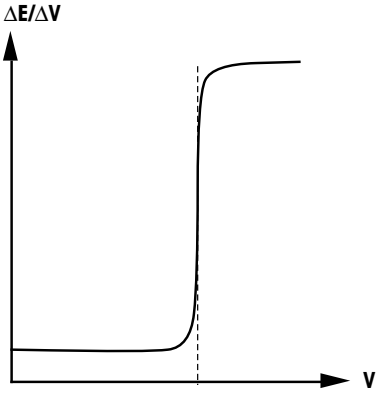
The break point of a linear titration curve can be obtained through extrapolation of the bordering straight parts of the curve and calculation of their intersection points.

The main problem here lies in finding parts of the curve that can be regarded as representative straight lines. Often only small sections of the titration curve are approximately linear. It should be noted that all measured values E_i must be corrected for dilution – multiplication of all E_i's by the factor $(V_0 + V_i)/V_0$. If this measure is omitted, even the linear sections are slightly curved.



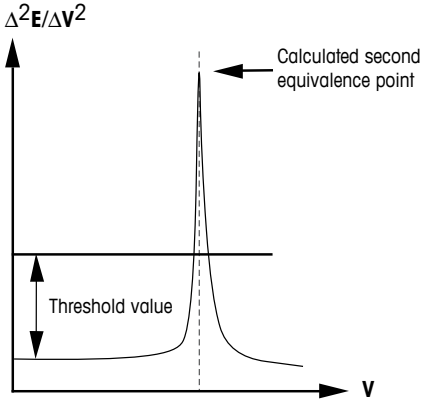
The following method serves as an alternative to the procedure involving straight line extrapolation:
The first derivative of a segmented curve shows the typical shape of an S-shaped curve whose inflection point is a good approximation of the equivalence point.

The evaluation of segmented curves is performed with the same procedure as for symmetrical S-shaped titration curves (cf. chapter 8.2.1), but the calculated data of the first derivative are used rather than the data points of the titration curve.

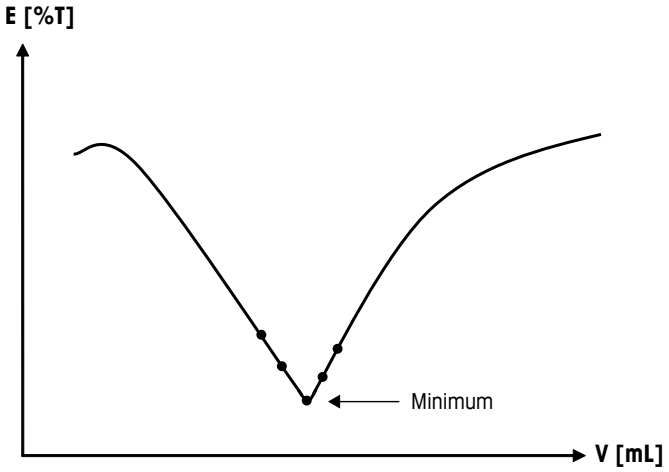


The equivalence point is thus recognized not with the aid of the calculated first derivative but by using the calculated second derivative. The threshold for the equivalence point recognition also refers to data of the second derivative.

The individual sections do not need to be exactly linear. Decisive for an exact determination of the equivalence point is the presence of a distinct break between the individual sections of the titration curve.



8.2.4 Titration curves with minimum and maximum values



This curve shows the typical profile obtained from turbidimetric titrations, for example, determination of the anionic surfactant content, where a colloidal precipitate is formed by adding the titrant. This precipitate leads to an increased turbidity of the solution. After the EQP the additional titrant dilutes the sample again, making the solution less opaque. The profile of the curve is therefore characterized by a minimum in the curve, which is the position of the equivalence point EQP.

The Excellence and Compact titrators provide the evaluation procedures 'Maximum' and 'Minimum'. An equivalence point is recognized when the greatest (smallest) potential value of the titration curve is greater (less) than two preceding and two subsequent values.

8.2.5 Measures for unambiguous equivalence point recognition

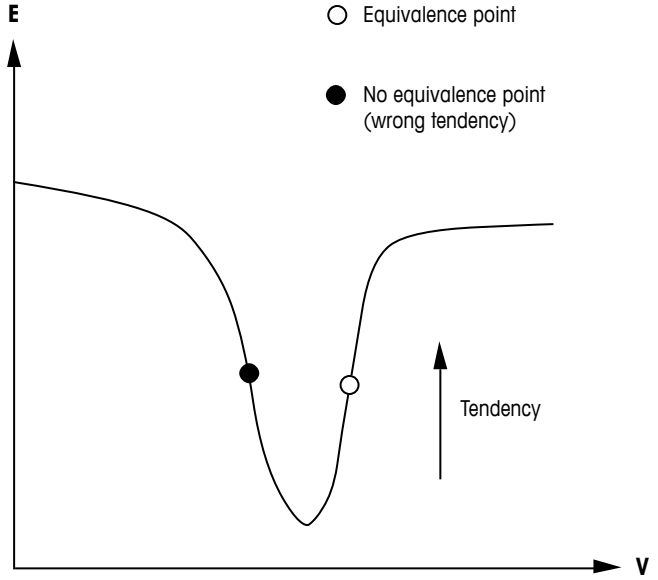
Symmetrical, asymmetrical and segmented titration curves

In order to enhance the reliability of correct equivalence point recognition the following evaluation procedures are available in the Compact and Excellence titrators:

- Specification of the tendency
- Specification of a threshold value
- Specification of an recognition point range
- Steepest jump
- Last EQP

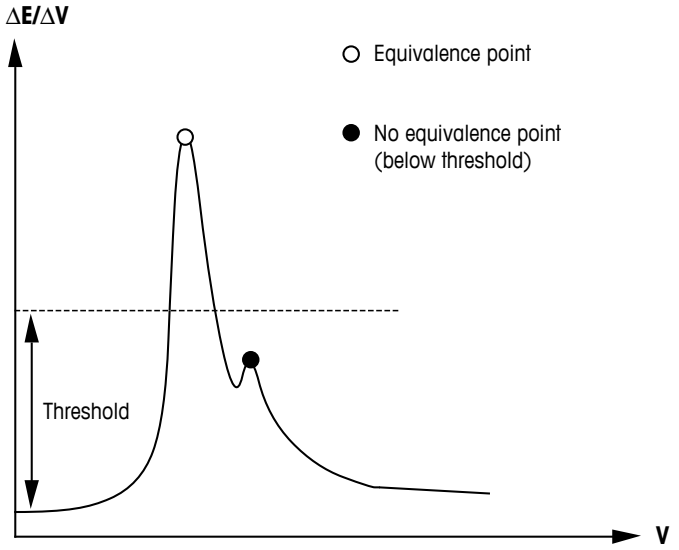
Tendency

The tendency defines the titration direction. This can thus be used to filter out all equivalence points whose titration direction is not in accordance with the tendency. The following illustration shows a schematic representation of a typical titration curve of a photometrically indicated surfactant determination. With the aid of the tendency, only one of the two equivalence points possible in principle will be evaluated.



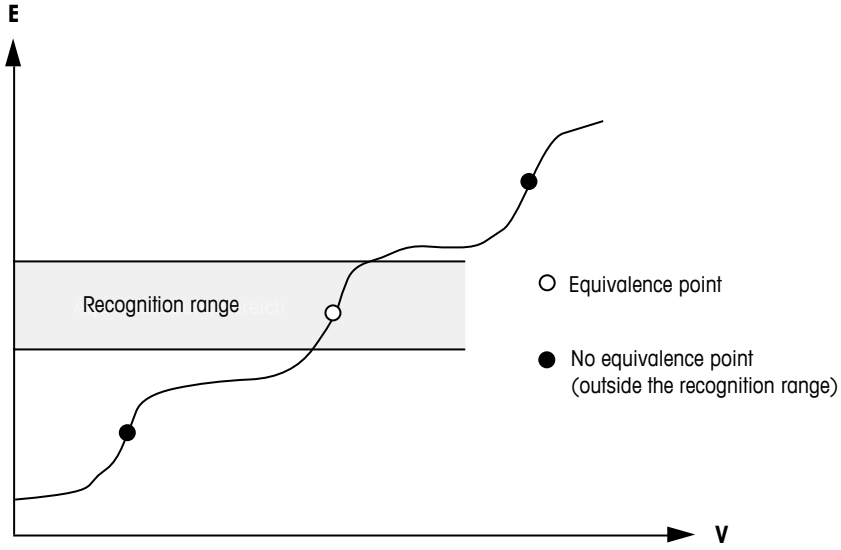
Threshold value

The threshold value allows suppression of flat jumps with S-shaped curves. All maxima in the first derivative that lie below the threshold value are ignored (see illustration).



Recognition Range

With the recognition range a measured value range is defined, within which the equivalence points must occur. All equivalence points outside this range are not evaluated (see illustration).



In the Excellence and Compact titrators a maximum of 9 recognition ranges can be defined.

Last EQP and Steepest Jump

The recognition parameters 'Last EQP' and 'Steepest Jump' are additional criteria that further enhance the degree of reliability in the recognition of equivalence points. They can be defined individually for each recognition range or for the overall recognition range, i.e. the complete titration curve.

- Last EQP: Only last equivalence points are considered
- Steepest jump: Only steepest jumps of the titration curve are considered.

Titration curves with minimum and maximum values

In addition to the threshold value and recognition range the following parameters are available either individually for each recognition range or for the overall recognition range, i.e. the complete titration curve.

Lowest value: Only the defined lowest values are considered. This parameter is only available if the evaluation procedure 'Minimum' has been selected.

Highest value: Only the defined highest values are considered. This parameter is only available if the evaluation procedure 'Maximum' has been selected.

Last EQP: Only last equivalence points are considered

8.2.6 GRAN evaluation: the mathematical linearization of titration curves

The idea behind this procedure is based on the mathematical transformation of a model function of the titration curve

$E = f(V)$

in order to obtain linear relationship between the new variables X and Y:

$E = f(V)$: $E \rightarrow Y$
$V \rightarrow X$: $V \rightarrow Y = A \cdot X + B$

Depending on the model, the equivalence point can then be calculated from one of the following quantities:

- Slope A
- Intercept B (Y axis)
- Intercept -B/A (X axis)

This procedure was first used by Gran [3] to evaluate acid-base titrations.

Example:
Titration of a strong acid (e.g. HCl) with a strong base (e.g. NaOH)
In aqueous solution HCl and NaOH are completely dissociated throughout the titration:

$HCl + H_2O \rightarrow H_3O^+ + Cl^-$
$NaOH \rightarrow Na^+ + OH^-$

The following titration reaction occurs:

$2H_2O \rightleftarrows H_3O^+ + OH^-$

The corresponding law of mass action (ionic product of water) is given by

$$K_w = \left[H_3O^+\right] \cdot \left[OH^-\right]$$

The charge balance is as follows:

$$[H_3O^+]+[Na^+]=[Cl^-]+[OH^-]$$

During the titration, the mass balances of the ions that do not participate in the titration reaction are:

$$[Na^+]=\frac{CONC \cdot V}{V_0+V}$$

$$[Cl^-]=\frac{CONC \cdot VEQ}{V_0+V}$$

with:

CONC:	concentration of the titrant NaOH	[mol/L]
V_0:	initial volume of the titration	[mL]
V:	volume of titrant added	[mL]
VEQ:	titrant consumption up to the equivalence point	[mL]

Insertion of the mass balances in the charge balance equation and solving for $[H^+]$ ($[H^+]=[H_3O^+]$ yields a quadratic equation:

$$[H^+]^2-[H^+]\cdot\frac{CONC \cdot (VEQ-V)}{V_0+V}-K_w=0$$

The principle of the mathematical linearization of the titration curve can be demonstrated by using the example above:

Starting from the charge balance equation

$$[OH^-] - [H^+] = [Na^+] - [Cl^-] = \frac{CONC \cdot (V - VEQ)}{V_0 + V}$$

Solving of the equation by consideration of G=VEQ-V and the ionic product of water yields:

$$G = VEQ - V = \frac{V_0 + V}{CONC} \left([H^+] - \frac{K_w}{[H^+]} \right)$$

Representation of G as a function of V gives the desired linear relation.

Before the equivalence point (V < VEQ)

$$G_1 = VEQ - V = \frac{V_0 + V}{CONC} \cdot [H^+]$$

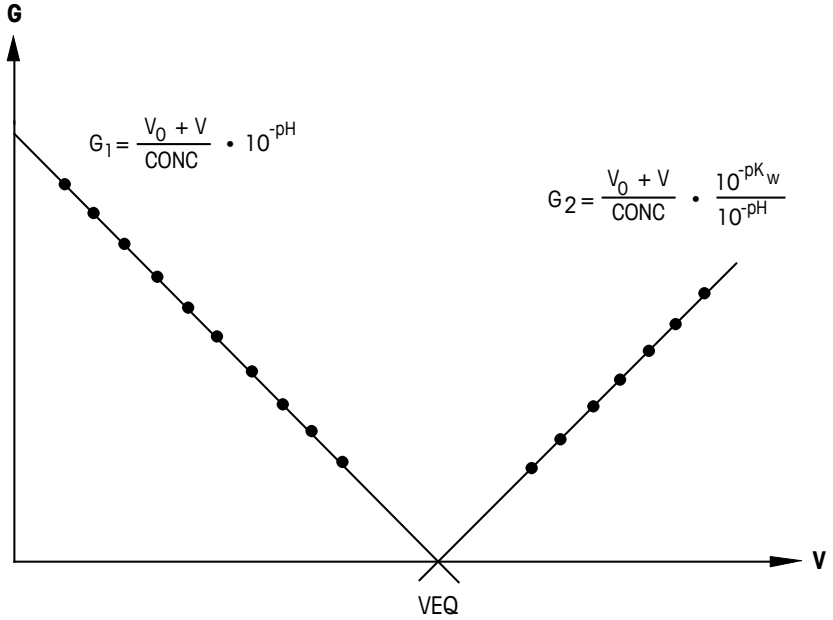
is valid.

After the equivalence point (V > VEQ)

$$G_2 = V - VEQ = \frac{V_0 + V}{CONC} \cdot \frac{K_w}{[H^+]}$$

is valid.

In the graphical representation of the transformed data, two straight lines are obtained with slopes of –1 and +1 that intersect the V axis at V = VEQ.



This graph, often called a Gran's plot, demonstrates the potential advantages of this method: There is no need to titrate to the equivalence point; it can be determined using a few experimental points either graphically or by calculation using linear regression. However, a requirement for this is that the sensor parameters (zero point and slope) and the start volume V_0 are known exactly. Otherwise, curves exhibiting partial curvature result and the determination of VEQ is inaccurate.

It should also be noted that for each titration reaction this method requires both an individual transformation function G and a knowledge of additional model parameters such as the stability constants.

8.3 The half neutralization value

The acidity constant K_a (see section 3.1.4) or the pK_a value is a measure of the strength of an acid in the particular solvent. The pK_a value is not only an important quantity in the classification of an acid, but also determines the properties of the substance in nature or its possible use as a drug.

The determination of exact values of the pK_a by means of titration is a demanding task. The correct procedure requires not only an exact knowledge of the sensor parameters, but also the use of activities rather than concentrations.

From the law of mass action of the reaction of an acid HA with H_2O (see section 3.1.4)

$$K_a = \frac{[H_3O^+] \cdot [A^-]}{[HA]}$$

it follows that

$$pH = pK_a + \log \frac{[A^-]}{[HA]}$$

When the acid HA is half neutralized, i.e. V = VEQ/2, the concentration [HA] of the still undissociated acid is approximately equal to the concentration $[A^-]$ of the base. Hence

$$pH = pK_a$$

This pH value at half consumption to the equivalence point is called the half neutralization value. It can easily be shown that this half neutralization value is a good estimation of the pK_a value of weak acids.

The acidity constant K_a can be calculated at any point during the titration with a strong base if all parameters are known [4]:

$$K_a = [H^+] \cdot \frac{c_T + d}{c_A - c_T - d}$$

with:

$$d = [H^+] - [OH^-]$$

$$c_T = \frac{CONC \cdot V}{V_0 + V}$$

$$c_A = \frac{CONC \cdot VEQ}{V_0 + V}$$

The quantity c_T is the concentration of the added titrant and c_A the concentration of acid in the titration vessel during the titration. At the half neutralization value (V = VEQ/2)

$$c_T = c_{HNV} = 0.5 \cdot c_A$$

and hence

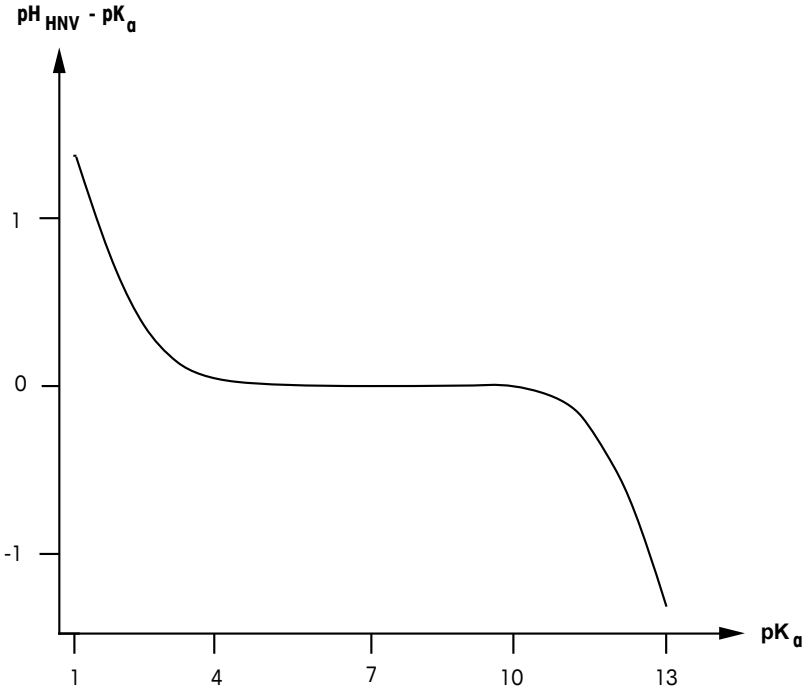
$$K_a = [H^+] \cdot \frac{c_{HNV} - d}{c_{HNV} + d}$$

or

$$pH_{HNV} = pK_a + \log \frac{c_{HNV} + d}{c_{HNV} - d}$$

If the condition $d << c_{HNV}$ is fulfilled, the half neutralization value pH_{HNV} is equal to the pK_a.

Example:
Difference between pH_{HNV} and pK_a in the titration of 50 mL of an acid of concentration 0.01 mol/L with 0.1 mol/L NaOH (VEQ = 5 mL)



As revealed in the figure above, for pK_a values between 4 and 10 the half neutralization value represents an excellent approximation of the pK_a value. For strong acids ($pK_a < 4$) the half neutralization value gives an overestimate of the pK_a value.

[1] C.F. Tubbs, Anal. Chem., 26, 1670 (1954)
[2] S. Ebel, E. Glaser, R. Kantelberg and B. Reyer, Fres. Z. Anal, Chem., 312, 604 (1982)
[3] G. Gran, Analyst, 77, 661 (1952)
[4] S. Ebel and W. Parzefall, "Experimentelle Einführung in die Potentiometrie", Verlag Chemie, Weinheim, Chapter 3 (1975).

9 Direct measurement, calibration

Besides titration, direct measurement of the concentration with a suitable sensor is the most frequently used method of determination in wet chemical analysis. This is especially true in the case of water analysis where not only measurement of the pH and the redox potential, but also the concentration determination with ion selective sensors and the determination of the conductivity, turbidity and oxygen content are important.

This section discusses the pH measurement, measurement of the redox potential, direct measurement with ion selective sensors and measurement of the conductivity. Particular attention is paid to the special aspects of sensor calibration.

9.1 pH measurement

The pH is defined as the negative logarithm of the hydrogen ion activity (see section 3.1.3). The range of the pH value is given by the autoprotolysis of water and lies between pH 0 and pH 14. The ionic product

$K_w = a_{H^+} \cdot a_{OH^-}$

and hence also the neutral point ($[H^+] = [OH^-]$) strongly depends on the temperature.

Temperature [°C]	K_w [mol²/L²]	Neutral point [pH]
0	$0.11 \cdot 10^{-14}$	7.47
25	$1.0 \cdot 10^{-14}$	7.00
50	$5.5 \cdot 10^{-14}$	6.63
100	$54.0 \cdot 10^{-14}$	6.13

For thermodynamic reasons it is not possible to calculate or measure the pH exactly. Various approximation methods and conventions have thus been developed [1]. The standard for the pH scale comprises different buffer solutions whose pH has been fixed by convention. The pH values of these so-called NIST (NBS) buffer solutions have also been accepted by DIN [2].

These buffer solutions are mixtures of substances with a stable hydrogen ion activity, that show little change on dilution or in the presence of impurities. The pH values of these standard buffer solutions are tabulated between 0 and 95 °C (see appendix A). The buffer solutions have the following pH values at a reference temperature of 25 °C:

1.679, 4.005, 6.865, 7.413, 9.180 and 10.012

For routine calibration many chemical producers and sensor manufacturers offer so-called technical buffer solutions with predominantly integral pH values. These are less susceptible to dilution and have greater buffer capacities than the NIST (NBS)/DIN buffers. METTLER TOLEDO [3] and MERCK [4] are among the companies that offer such buffer solutions. The temperature dependencies of the technical buffer solutions of both producers are very similar.

Tables of the temperature dependencies of these technical buffer solutions from METTLER TOLEDO and MERCK can be found in appendix A.

The buffer solutions with pH values 4.60 and 7.00 correspond to the zero points of commercial glass sensors. For routine calibration of the glass sensors, buffer solutions of pH 4, 7 and 10 are normally used.

9.1.1 Calibration of a pH sensor

The potential of a pH sensor assembly is described by the Nernst equation:

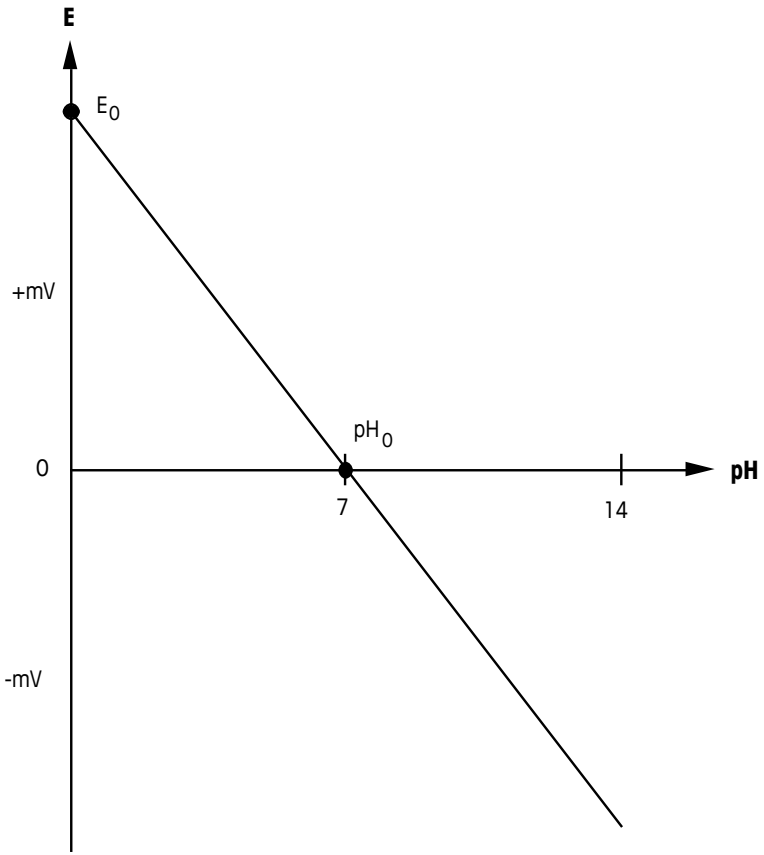
$$E = E_0 + S \cdot pH$$

E_0 is the standard potential at pH = 0. S is the slope and defines the change in potential per pH unit. The slope is temperature dependent.

$$S = -2.301 \frac{R \cdot T}{F}$$

Temperature [°C]	Slope S
0	–54.20
25	–59.16
50	–64.12

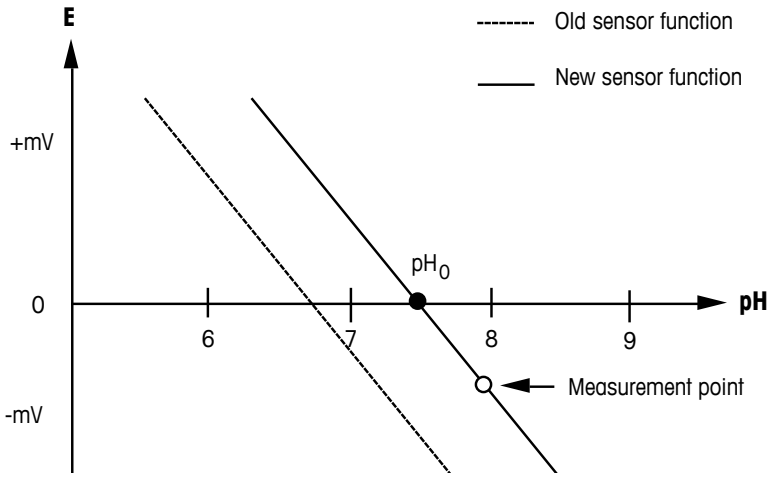
The sensor zero point pH_0 (pH value at E = 0 mV) and the slope S are normally specified as calibration parameters.



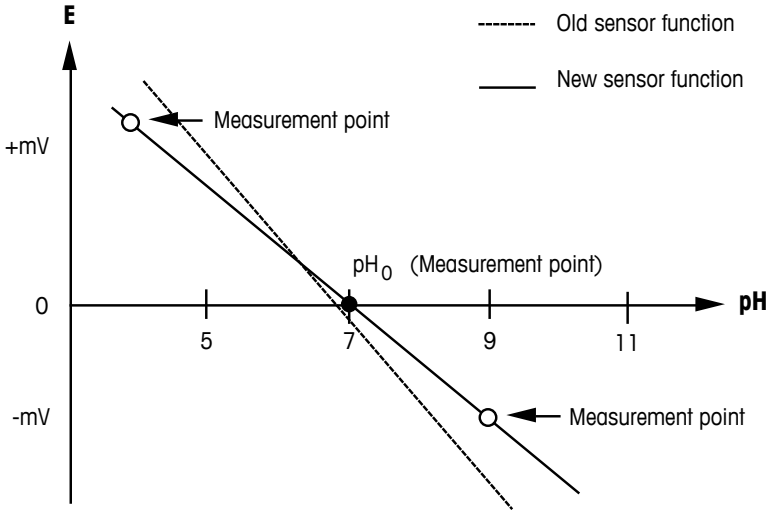
The combination glass sensors used in routine analysis have an sensor zero point at pH 7. New sensors should have a slope greater than 97% of the theoretical slope predicted by the Nernst equation.

Both calibration parameters vary slightly from sensor to sensor. For accurate pH measurements a calibration is thus necessary. The calibration can be performed as a single- or multipoint measurement.

If the pH is measured over a narrow range, as a rule a single-point calibration suffices. Here, only the sensor zero point is redetermined, the slope is not checked.



The multipoint calibration utilizes two or more buffer solutions whose pH value must differ by at least one pH unit. The new values for the calibration parameters pH_0 and S are obtained by linear regression through the measured points.

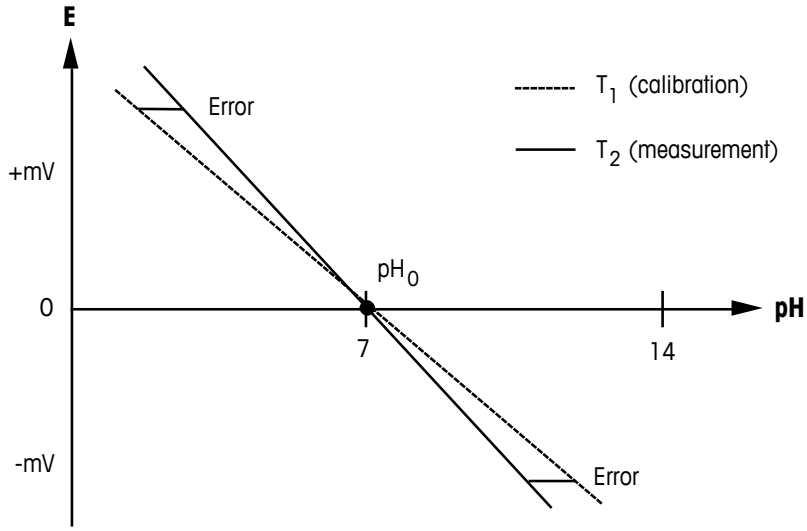


9.1.2 Temperature compensation

If a pH measurement is performed at a temperature different from that of the calibration, an error must be anticipated.

It becomes greater

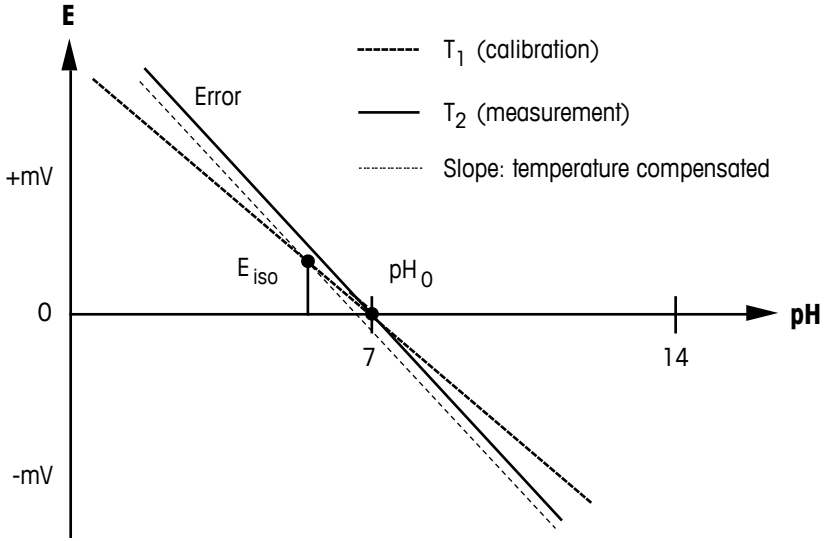
- with increasing difference between measurement and calibration temperature and
- with increasing difference between the pH of the analysis solution and the sensor zero point (~ pH 7).



The error can be rectified if the slope is temperature corrected:

$$S(T_2) = S(T_1) \cdot \frac{T_2 + 273.16}{T_1 + 273.16}$$

This correction is only approximately correct since the intersection point of the calibration lines at different temperatures, the so-called isothermal intersection point E_{iso} does not lie at exactly 0 mV [5]. Even though the slope has been corrected for temperature, an error still arises.



This error increases with increasing temperature difference between calibration and measurement and with increasing value of E_{iso}. The error is independent of the pH of the solution.

The isothermal intersection point can be determined by calibration with two buffer solutions at two different temperatures graphically or by calculation.

Temperature compensation which takes into account the isothermal intersection point is then free from error when the temperature behavior of the slope follows the Nernst equation. This may not necessarily be the case. For precise measurements a previous calibration at the desired measurement temperature is thus always advisable.

9.2 Direct measurement with ion selective sensors

The potential of an ion selective sensor depends on the ionic activity in the solution and like that of the pH sensor is described by the Nernst equation:

$$E = E_0 + S \cdot p_A$$

p_A is the negative logarithm of the ionic concentration of entity A. The negative logarithm of the ionic concentration of cations is designated pM, that of anions pX.

In this notation the sign of the slope S for cations is negative and that for anions positive.

$$S = \pm 2.301 \frac{R \cdot T}{n \cdot F}$$

The theoretical slope of an ion selective sensor at 25 °C is 59.16 mV for monovalent ions and 29.58 mV for bivalent ions.

The following criteria must be heeded when working with ion selective sensors:

Sample pretreatment
Sample pretreatment is the most important factor in direct measurement with ion selective sensors. For quantitative analyses each sample solution must be mixed with a certain amount of electrolyte solution (so-called TISAB solution). These buffer solutions can perform the following functions:

- Ensure constant ionic strength of all sample and calibration solutions. With this measurement technique the ion selective sensor can be used for direct measurements of the analytical concentration and not just the ionic activity.
- pH buffering: Depending on the application, the medium must be acidic, neutral or basic.
- Elimination of interfering ions through complexation, oxidation or reduction.
- Destruction of all complexes with the analyte ion to determine the total ionic concentration.

Selectivity

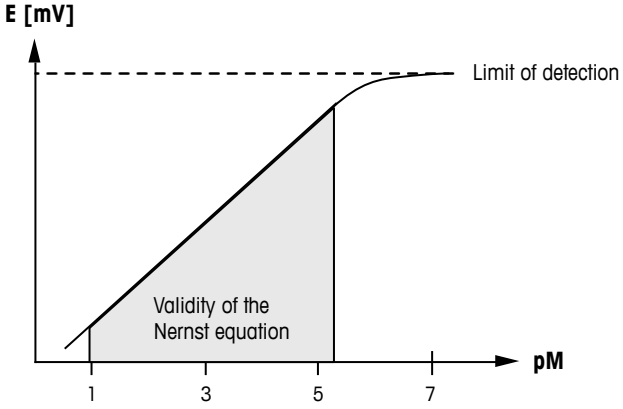
The selectivity of an ion selective sensor is limited and is expressed by the selectivity coefficient (see Section 4.1.5). Interferences due to foreign ions can be suppressed by appropriate sample pretreatment.

Precision

In the ideal case, the attainable precision for univalent ions is 1 to 2%, that for bivalent ions 2 to 4%.

Limit of detection

The lower limit of detection is determined by the ions released by the sensor itself. This minimum concentration depends on the solubility of the active membrane substance in the medium. In the vicinity of the limit of detection the potential deviates from the linear behavior predicted by the Nernst equation.



Calibration

In order to perform a direct measurement the sensor must be calibrated with solutions of known concentration. In the linear region two calibration points are sufficient, but in the nonlinear region several points are necessary.

9.3 Redox measurement

The determination of the redox potential with a redox sensor is also a potentiometric measurement. The redox potential is a measure of how easily a substance can accept or donate electrons.

The measurable redox potential follows the Nernst equation. The correct equation is obtained from the oxidation or reduction process.

Examples:

a. $Fe^{2+} \rightarrow Fe^{3+} + e^-$

$$E = E_0 + 2.301 \frac{R \cdot T}{F} \log \frac{\left[Fe^{3+}\right]}{\left[Fe^{2+}\right]}$$

b. $Mn^{2+} + 4H_2O \rightarrow MnO_4^- + 8H^+ + 5e^-$

$$E = E_0 + 2.301 \frac{R \cdot T}{5 \cdot F} \log \frac{\left[MnO_4^-\right]\left[H^+\right]^8}{\left[Mn^{2+}\right]}$$

These examples show that the redox potential is always determined by the ratio of the oxidizing and reducing components and can also be pH dependent. The oxidizing or reducing action of an analysis solution can thus also be a function of the pH.

Measurement of the redox potential is not carried out all that often for the following reasons:

- difficulties in the interpretation of the results
- measurement difficulties

In water analysis the redox potential is a frequently determined measured value (e.g. analysis of the water quality of swimming pools).

Calibration of redox sensors is not necessary since in contrast to pH sensors no changes in the zero point and slope occur. Wrong redox potentials can be traced to a contaminated sensor surface.

The use of redox buffer solutions is thus restricted to a purely operational test of the redox sensor.

A temperature compensation is not needed with redox sensors. The measurement temperature must still be specified, however, since the temperature coefficient of the redox potential can be very large.

9.4 Conductivity measurement

The determination of the conductivity [6], [7], [8] as a measure of how well a solution conducts an electric current has already been discussed in section 4.3.1. In addition to its use as an indication method for titrations, direct determination of the conductivity has achieved an important standing. The most important application areas of the conductivity measurement are:

- purity check of bodies of water
- purity check of nonaqueous solutions
- concentration determinations
- monitoring of baths (e.g. in electroplating)
- process analysis

The magnitude of the measured conductivity χ depends on the concentrations of all charged particles in the solution. Typical conductivity values are:

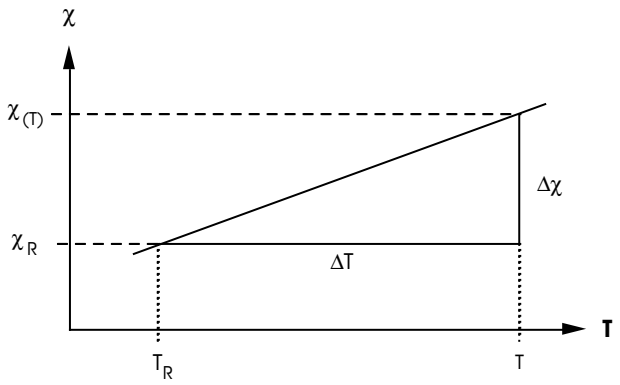
distilled water	0.1–10 µS/cm
drinking water	100–1000 µS/cm
waste water	1–10 mS/cm
sea water	1–100 mS/cm
conc. acids and bases	100–1000 mS/cm

9.4.1 Calibration and temperature compensation

Accurate conductivity measurements demand a temperature compensation since the conductivity of an aqueous solution increases with increasing temperature. Conductivity values are thus specified only together with the measurement temperature or corrected by calculation to a reference temperature (usually 25 °C).

The temperature dependency of the conductivity is described by the temperature coefficient α. The value of α is defined as the change in the conductivity χ when the temperature changes by 1 °C referred to the conductivity χ_R at a reference temperature T_R.

$$\alpha = \frac{\Delta\chi}{\Delta T}\,\frac{1}{\chi_R} \cdot 100\%$$



For temperature ranges within ±10 °C of the value of the reference temperature, the temperature coefficient is virtually constant. The measured conductivity is corrected to the reference temperature by means of the following equation:

$$\chi_R = \frac{G(T) \cdot Z}{1 + \frac{\alpha}{100}(T - T_R)}$$

with

χ_R : conductivity at the reference temperature T_R [µS/cm, mS/cm]
G (T): measured conductance at temperature T [µS, mS]
Z: cell constant [cm^{-1}]
a: temperature coefficient [%/ °C]
T_R: reference temperature [°C]
T: measurement temperature [°C]

The temperature coefficient a has a typical value of 0–4%/ °C. A mean value of 2%/ °C is frequently assumed.

For highly precise work the exact temperature dependence must be determined with the aid of a reference solution.

The exact determination and checking of the cell constants is effected using calibration solutions. The standards employed are KCl solutions of concentration 0.01, 0.1 and 1 mol/L.

[1] R.G. Bates, "Determination of pH", John Wiley (1973)
[2] DIN standard 19266
[3] „pH Theory Guide", Mettler-Toledo AG, Analytical, 2007
[4] MERCK booklet: Laboratory products for practical applications, 1987
[5] „pH Theory Guide", Mettler-Toledo AG, Analytical, 2007, chapter 4.6
[6] F. Oehme, "Angewandte Konduktometrie", Hüthig Verlag, Heidelberg (1961)
[7] F. Oehme and R. Bänninger, "ABC der Konduktometrie", offprint, Chemische Rundschau (1979)
[8] E. Pungor, "Oscillometry and Conductometry", Pergamon Press, Oxford (1965)

10 Assessment of the result

The aim of a titration is normally the determination of the content of a substance in a sample. The result of the analysis is used to assess the test sample.

In practice, each analytical result is associated with random and systematic errors. Every analyst must therefore always ask himself whether the quality of his titration results is good enough. The methods offered by statistics are an excellent means to assess the accuracy of the results ([1], [2], [3]).

10.1 Fundamentals of statistics

For the practical application of statistics in the context of titration, a brief description of the most important concepts and a listing of the associated calculation formulae will suffice here.

Arithmetic mean $\bar{x}$

The arithmetic mean $\bar{x}$ is equal to the sum of the independent measurement results x_i of a series of measurements divided by the number of measurements N.

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The mean is by far the most important quantity in statistics.

Variance s^2

The variance s^2 of a series of measurements is the sum of the squares of the deviations of the N individual values x_i from the arithmetic mean $\bar{x}$ divided by the number of degrees of freedom f (f = N-1).

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

Standard deviation s

The standard deviation s of a series of individual values x_i is a measure of the spread of the individual values x_i around the mean $\bar{x}$. It is given by the positive square root of the variance s^2.

The standard deviation has the unit of the individual values and should always be specified with one significant figure more than the mean.

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

For a duplicate determination, the standard deviation is given by

$$s = ½|x_1 - x_2|\sqrt{2}$$

Relative standard deviation (coefficient of variation)

The relative standard deviation s_{rel} or coefficient of variation CV of a series of individual values x_i is given by the standard deviation s divided by the mean $\bar{x}$.

$$s_{rel} = \frac{s}{\bar{x}}$$

In many cases a percentage value is preferred:

$$s_{rel} = \frac{s}{\bar{x}} \cdot 100[\%]$$

Number of degrees of freedom

The number of degrees of freedom f is the number of independent, individual values used in a calculation minus the number of quantities entering the calculation that have already been derived from such values.

Example:
For the calculation of the standard deviation s of N independent individual values, the number of degrees of freedom is $N-1$ since the arithmetic mean $\bar{x}$ also enters the calculation.

Confidence level P

The confidence level P denotes the probability that a certain statement is correct. In routine analysis a confidence level of 95% is normally used, whereas scientific investigations employ a level of 99%.

Normal distribution, t distribution

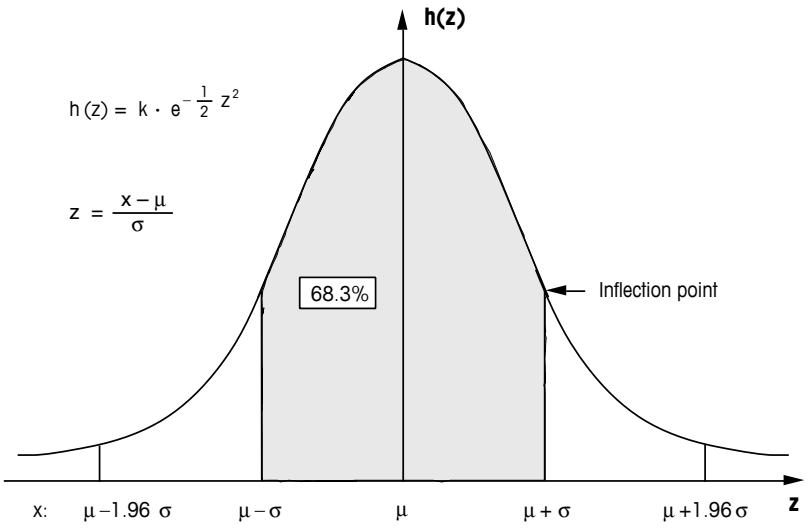
If a very large number of measured values is available, this is known as a population. If the frequency of the appearance h(x) of all values x is plotted against their size, the so-called Gaussian or normal distribution is obtained.

The value μ is the true mean value that identifies the position of the distribution. The scatter s of the population is a measure of the width of the distribution.

$$\mu = \lim_{n \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\sigma = \lim_{n \to \infty} \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

Gaussian or normal distribution



Within the limits of $\mu \pm \sigma$, 68.3% of all measured values are expected (hatched area), whereas 95% of all measured values lie between $\mu - 1.96\ \sigma$ and $\mu + 1.96\ \sigma$.

The spread T signifies that P% of all measured values are to be expected in the range $x + T$ and $x - T$. For P = 95%, the spread is given by

$$T = \pm 1.96\ \sigma$$

or in general

$$T = \pm \mu \cdot \sigma$$

The factor μ is known as the fractile of the normal distribution and depends only on the selected confidence level P.

If a small number of measured values is present (typical in analytical practice), this is called a random sample. If μ and s of the population are unknown, the distribution of the random sample values does not yet follow the normal distribution. The mean $\bar{x}$ is the best estimate of the true value but is still associated with a statistical uncertainty that needs to be taken into account. The distribution of the random sample will thus be broader than the normal distribution. The theoretical function of this distribution is called the t or student distribution.

The spread T of the individual measurement and the confidence interval CI of the mean value can thus be calculated.

$$T = \pm s \cdot t$$

$$CI = \pm \frac{s \cdot t}{\sqrt{N}}$$

The confidence interval CI (= spread of the mean) signifies that the true mean lies within the range $\bar{x}$ + CI and $\bar{x}$ -CI with P% certainty.

The factor t is the fractile of the t distribution and depends on the confidence level P and the number of measured values N; it can be taken from the t table (see appendix B).

Outliers, Grubbs outlier test

If a series of measurements (N > 3) includes one or more values that deviate widely from the mean, the question must be asked whether these measured values are indeed correct or whether they should be considered outliers. Outliers have the following influence on the overall result of an analysis:

- the mean value is clearly shifted to a higher or a lower value
- the standard deviation is increased
- the distribution of the individual values about the mean is distorted and no longer follows a normal distribution.

Such outliers are uncovered by means of statistical tests, e.g. with the Grubbs test. Here, the mean $\bar{x}$ and the standard deviation s of the analysis data are first calculated. From the experimental data the value x* with the greatest deviation from the mean is sought and tested using the following condition equation:

$$TV = \frac{\left| x^* - \bar{x} \right|}{S}$$

The test variable TV is compared with the value in the Grubbs table G (N, P%), which in turn depends on the number of analysis values N. A Grubbs table can be found in appendix B.

If the test variable TV is greater than G (N, P%), the experimental value under test is considered an outlier and deleted from the series of measurements. The remaining data of the series are used to calculate new values of the mean, and the standard deviation and the outlier test repeated for another value suspected of being an outlier.

Example:
A content determination (N = 6) gave the following results:

x_1 = 30.38%
x_2 = 30.23%
x_3 = 30.34%
x_4 = 29.98%
x_5 = 30.29%
x_6 = 30.31%

$\bar{x}$ = 30.26%
s = 0.144%

outlier suspect: x_4

$$TV = \frac{|x_4 - \bar{x}|}{S} = 1.944$$

G(6.90%) = 1.822 → x_4 is an outlier.

After removal of x_4 the sample is free from outliers.

10.2 Concepts relating to correctness

The deviation from the correct value or that value accepted as correct of an analytical result is known as the error.

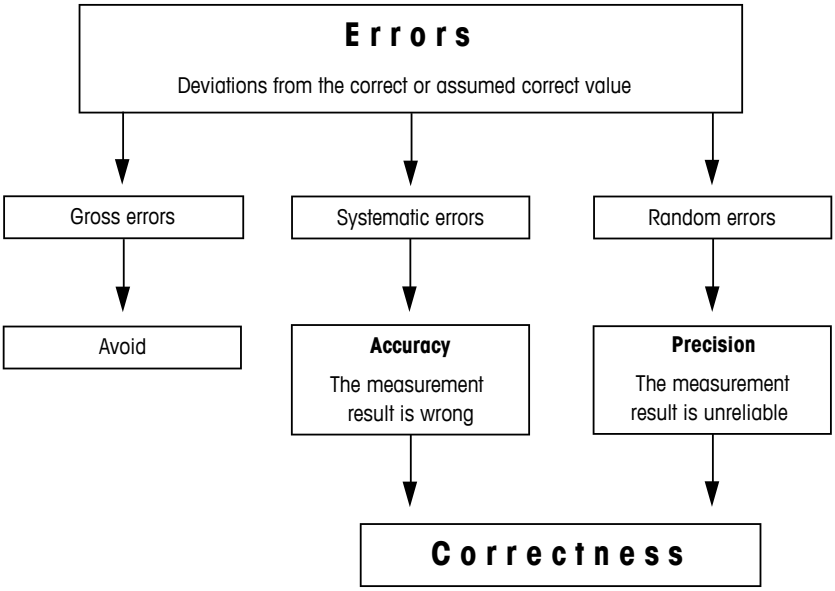
Gross errors
Gross errors arise through failure to follow analytical procedures, through faulty analytical instruments and through carelessness on the part of the analyst. From the statistics point of view, gross errors are always avoidable.

Random errors
Random errors are as a rule unavoidable and are responsible for the spread in the individual measured values about the mean. The magnitude of the random errors – determined using the standard deviation s – is a measure of the precision of an analytical procedure.

Systematic errors
Systematic errors give rise to a difference between the expected value and the mean $\bar{x}$ obtained from the measurement and thus determine the accuracy of an analytical procedure. Systematic deviations have a definite cause and in principle can be rectified. A distinction is made between proportional systematic and constant systematic deviations. It is apparent from the standard formula for the calculation of titration results

$$R = C \cdot (VEQ \cdot CONC - BLANK)/m$$

that, for example, a wrong titrant concentration will lead to a proportional systematic error and a wrong solvent blank value to a constant systematic error.



The correctness of an analysis method is a qualitative concept. It describes the systematic and random deviations of the measurement results from the true value. The accuracy (determined by systematic errors) and the precision (determined by random errors) are subdivisions of the concept of correctness.

A titration comprises a pretreatment part and the analytical process with the titrator. In the first part the analyst has a great influence on the error by way of his method of working. In the actual measurement process (s)he has no influence on the correctness of the results. Even assuming correct maintenance by the user, the quality of the analytical result of the titration is still greatly dependent on the quality of the instrument. This is the responsibility of the manufacturer.

10.3 Limit of detection, limit of determination

The limit of detection and the limit of determination are two concepts that are constantly confused.

The limit of detection describes the smallest amount of substance or lowest concentration that can be distinguished qualitatively from zero amount of substance with a specified confidence level (e.g. 95%) in a single analysis.

The limit of determination is the smallest amount of substance or lowest concentration that can be determined quantitatively with a specified confidence level (e.g. 95%) and can be distinguished from zero amount of substance. The limit of determination provides no information regarding the correctness of the analytical procedure.

10.4 Standard, standard samples, control samples

A standard is a chemical substance that is used as a reference sample. It is used to check the accuracy of an analytical method.

Standard samples are samples containing a component whose concentration is known with sufficient accuracy and which can be used as standards.

Control samples are analysis samples whose matrix composition corresponds very closely to that of real samples. They are used to check the accuracy and the precision of an analysis method.

10.5 Consequences for practical application

To guard against systematic errors in a titration, several measures need to be implemented:

- Regular determination of the titer of the titrant used (also applies when commercial volumetric solutions are used)
- Determination of any blank value of the solvent or the matrix
- Regular use of control samples.

For the standardization (titer determination) and control of titrimetric solutions, substances are needed that within the limits of measurement accuracy of the titration (lower limit: 0.01%) correspond exactly to the composition given by their formula. These so-called primary standards must have the following properties:

- Clearly defined composition and high degree of purity
- Large molar mass (avoidance of weighing errors)
- Capable of being weighed accurately without difficulties (insensitive to oxygen and/or CO_2 and not hygroscopic)
- Concentration of a freshly prepared titrimetric solution remains stable
- Rapidly and easily soluble in the solvents needed
- Rapid and stoichiometric titration reaction.

Determinations of the titer and blank should be performed as a multiple determination (determinations in triplicate have proved adequate).

In routine analysis (use of the same analysis procedure for many samples), the regular insertion of control samples of known composition (e.g. primary standards) between the individual series has proved useful. This provides a check not only on the titer but also of the correct functioning of the analytical instrument. In addition to the result of the titration with the control sample, its progress (titration time, titration curve, etc.) provides indications of any problems (contamination of the sensor, etc.).

Note:
Tables of primary standards for the most important titrants can be found in appendix C.

[1] J.C. Miller and J.N. Miller, "Statistics for Analytical Chemistry", second edition, Ellis Horwood, Chichester, 1988.
[2] R. Caulcutt and R. Boddy, "Statistics for Analytical Chemists", Chapman and Hall, London, 1983.
[3] W. Funk, V. Damman, C. Vonderheid und G. Oehlmann, "Statistische Methoden der Wasseranalytik", VCH Verlagsgesellschaft, Weinheim, 1985

Appendices

Appendix A:
Tables showing the pH temperature dependencies of DIN/NIST (NBS), MERCK and METTLER TOLEDO buffers (see section 9.1)

Appendix B:
Statistical tables (see section 10.1)

Appendix C:
Tables of primary standards for the most important titrants (see section 10.5)

METTLER TOLEDO EU buffers (Ref. 25°C)

Temperature [°C]	pH						
0	2.03	4.01	4.66	7.12	9.52	10.65	11.90
5	2.02	4.01	4.65	7.09	9.45	10.52	11.72
10	2.01	4.00	4.64	7.06	9.38	10.39	11.54
15	2.00	4.00	4.63	7.04	9.32	10.26	11.36
20	2.00	4.00	4.62	7.02	9.26	10.13	11.18
25	**2.00**	**4.01**	**4.60**	**7.00**	**9.21**	**10.00**	**11.00**
30	1.99	4.01	4.61	6.99	9.16	9.87	10.82
35	1.99	4.02	4.62	6.98	9.11	9.74	10.64
40	1.98	4.03	4.63	6.97	9.06	9.61	10.46
45	1.98	4.04	4.64	6.97	9.03	9.48	10.28
50	1.98	4.06	4.66	6.97	8.99	9.35	10.10
55	1.98	4.08	4.67	6.98	8.96	9.22	9.92
60	1.98	4.10	4.69	6.98	8.93	9.09	9.74
70	1.99	4.16	4.71	7.00	8.88	8.96	9.56
80	2.00	4.22	4.73	7.04	8.83	8.83	9.38
90	2.00	4.30	4.75	7.09	8.79	8.70	9.20
95	2.00	4.35	4.77	7.12	8.77	8.57	9.02

METTLER TOLEDO USA buffers (Ref. 25 °C)

Temperature [°C]	pH			
5	1.67	4.01	7.09	10.25
10	1.67	4.00	7.06	10.18
15	1.67	4.00	7.04	10.12
20	1.68	4.00	7.02	10.06
25	**1.68**	**4.01**	**7.00**	**10.01**
30	1.68	4.01	6.99	9.97
35	1.69	4.02	6.98	9.93
40	1.69	4.03	6.97	9.89
45	1.70	4.04	6.97	9.86
50	1.71	4.06	6.97	9.83

DIN (19266) / NIST buffers (Ref. 25 °C)

Temperature [°C]	pH								
0	1.666	3.577	3.863	4.010	6.984	7.534	9.464	10.317	13.423
5	1.668	3.573	3.837	4.004	6.950	7.502	9.392	10.248	13.207
10	1.670	3.569	3.819	4.001	6.922	7.474	9.331	10.180	13.003
15	1.672	3.565	3.801	4.001	6.900	7.451	9.277	10.121	12.810
20	1.676	3.561	3.787	4.003	6.880	7.432	9.228	10.066	12.627
25	**1.680**	**3.557**	**3.775**	**4.008**	**6.865**	**7.416**	**9.184**	**10.014**	**12.454**
30	1.685	3.553	3.766	4.015	6.853	7.405	9.144	9.970	12.289
35	1.691	3.549	3.759	4.026	6.845	7.396	9.110	9.928	12.133
40	1.697	3.549	3.754	4.036	6.837	7.389	9.076	9.892	11.984
45	1.704	3.544	3.751	4.049	6.834	7.386	9.046	9.856	11.841
50	1.712	3.548	3.748	4.064	6.833	7.384	9.018	9.830	11.705
55	1.715	3.554	3.750	4.075	6.834	7.382	8.985	9.804	11.574
60	1.723	3.560	3.753	4.091	6.836	7.380	8.962	9.778	11.449
70	1.743	3.580	3.763	4.126	6.845	7.378	8.921	9.752	11.324
80	1.766	3.609	3.780	4.164	6.859	7.376	8.885	9.726	11.199
90	1.792	3.650	3.802	4.205	6.877	7.374	8.850	9.700	11.074
95	1.806	3.674	3.815	4.227	6.886	7.372	8.833	9.674	10.949

MERCK buffers (Ref. 20 °C)

Temperature [°C]	pH							
0	0.96	2.01	3.05	4.05	4.68	5.06	6.04	6.98
5	0.99	2.01	3.05	4.04	4.68	5.05	6.02	6.95
10	0.99	2.01	3.03	4.02	4.67	5.02	6.01	6.92
15	0.99	2.00	3.01	4.01	4.67	5.01	6.00	6.90
20	**1.00**	**2.00**	**3.00**	**4.00**	**4.66**	**5.00**	**6.00**	**6.88**
25	1.01	2.00	3.00	4.01	4.66	5.00	6.02	6.86
30	1.01	2.00	3.00	4.01	4.66	5.00	6.03	6.86
35	1.01	2.00	3.00	4.01	4.66	5.00	6.03	6.85
40	1.01	2.00	2.98	4.01	4.67	5.00	6.04	6.84
45	1.01	2.00	2.98	4.01	4.67	5.01	6.05	6.84
50	1.01	2.00	2.97	4.00	4.68	5.01	6.06	6.84
60	1.02	2.00	2.97	4.00	4.69	5.04	6.10	6.84
70	1.02	2.01	2.97	4.00	4.70	5.05	6.12	6.84
80	1.02	2.01	2.97	4.00	4.71	5.10	6.17	6.86
90	1.02	2.01	2.96	4.00	4.72	5.14	6.24	6.88

MERCK buffers (Ref. 20 °C) (continued)

Temperature [°C]	pH							
0	7.13	8.15	9.24	9.46	10.26	11.45	12.58	13.80
5	7.07	8.10	9.16	9.40	10.17	11.32	12.41	13.59
10	7.05	8.07	9.11	9.33	10.11	11.20	12.26	13.37
15	7.02	8.04	9.05	9.28	10.05	11.10	12.10	13.18
20	**7.00**	**8.00**	**9.00**	**9.22**	**10.00**	**11.00**	**12.00**	**13.00**
25	6.98	7.96	8.95	9.18	9.94	10.90	11.88	12.83
30	6.98	7.94	8.91	9.14	9.89	10.81	11.72	12.67
35	6.96	7.92	8.88	9.10	9.84	10.72	11.67	12.59
40	6.95	7.90	8.85	9.07	9.83	10.64	11.54	12.41
45	6.95	7.88	8.82	9.04	9.79	10.56	11.44	12.28
50	6.95	7.85	8.79	9.01	9.74	10.48	11.33	12.15
60	6.96	7.83	8.73	8.96	9.67	10.33	11.04	11.75
70	6.96	7.80	8.70	8.93	9.62	10.19	10.90	11.61
80	6.97	7.78	8.66	8.89	9.55	10.06	10.70	11.39
90	7.00	7.75	8.64	8.85	9.49	9.93	10.48	11.15

1. t table

f	P = 95%	P = 99%	P = 99.9%
1	12.706	63.657	636.619
2	4.303	9.925	31.598
3	3.182	5.841	12.924
4	2.776	4.604	8.610
5	2.571	4.032	6.869
6	2.447	3.707	5.959
7	2.365	3.449	5.408
8	2.306	3.355	5.041
9	2.262	3.250	4.781
10	2.228	3.169	4.587
11	2.201	3.106	4.437
12	2.179	3.055	4.318
13	2.160	3.016	4.221
14	2.145	2.977	4.140
15	2.131	2.947	4.073
16	2.120	2.921	4.015
17	2.110	2.898	3.965
18	2.101	2.878	3.922
19	2.093	2.861	3.883
20	2.086	2.845	3.850
21	2.080	2.831	3.819
22	2.074	2.819	3.792
23	2.069	2.807	3.767
24	2.064	2.797	3.745
25	2.060	2.787	3.725
26	2.056	2.779	3.707
27	2.052	2.771	3.690
28	2.048	2.763	3.674
29	2.045	2.756	3.659
30	2.042	2.750	3.646
∞	1.960	2.576	3.291

2. Grubbs table: G (N, P%)

$P_{(one\text{-}tailed)}$	90%	95%	99%
N			
3	1.148	**1.153**	1.155
4	1.425	**1.463**	1.492
5	1.602	**1.672**	1.749
6	1.729	**1.822**	1.944
7	1.828	**1.938**	2.097
8	1.909	**2.032**	2.221
9	1.977	**2.110**	2.323
10	2.036	**2.176**	2.410
11	2.088	**2.234**	2.485
12	2.134	**2.285**	2.550
13	2.175	**2.331**	2.607
14	2.213	**2.371**	2.659
15	2.247	**2.409**	2.705
16	2.279	**2.443**	2.747
17	2.309	**2.475**	2.785
18	2.335	**2.504**	2.821
19	2.361	**2.532**	2.854
20	2.385	**2.557**	2.884
21	2.408	**2.580**	2.912
22	2.429	**2.603**	2.939
23	2.448	**2.624**	2.963
24	2.467	**2.644**	2.987
25	2.486	**2.663**	3.009
26	2.502	**2.681**	3.029
27	2.519	**2.698**	3.049
28	2.534	**2.714**	2.068
29	2.563	**2.745**	3.103
$P_{(two\text{-}tailed)}$	80%	90%*)	98%

Example:
G (6.90%) = 1.822

*) G (N, 90%) is used for the outlier test in the Excellence titrators.

Neutralization Titrations

Name	Formula	Solvent	Control	Indication	Standard Substance
Alkalimetry					
Sodium hydroxide	NaOH	deion. H_2O	2, 5	DG(i)111-SC, DG(i)115-SC	Potassium hydrogen phthalate
Potassium hydroxide	KOH	C_2H_5OH or CH_3OH or $(CH_3)_3COH$ or $i\text{-}C_3H_7OH$	2, 5	DG(i)113-SC, DGi116-Solvent	Benzoic acid
TBAH	$C_{16}H_{37}NO$	$CH_3OH/i\text{-}C_3H_7OH$	2, 5	DG(i)113-SC, DGi116-Solvent	Benzoic acid
Sodium methylate	$NaOCH_3$	CH_3OH	1, 5	DG(i)113-SC/DGi116-Solvent	Benzoic acid
Acidimetry					
Hydrochloric acid	HCl	deion. H_2O	3	DG(i)111-SC, DG(i)115-SC	Tris(hydroxymethyl)-aminomethane
		C_2H_5OH or C_3H_7OH	2	DG(i)113-SC/DGi116-Solvent	Tris(hydroxymethyl)-aminomethane
Sulfuric acid	H_2SO_4	deion. H_2O	3	DG(i)111-SC, DG(i)115-SC	Tris(hydroxymethyl)-aminomethane
Nitric acid	HNO_3	deion. H_2O	3	DG(i)111-SC, DG(i)115-SC	Tris(hydroxymethyl)-aminomethane
Perchloric acid	$HClO_4$	Glacial acetic acid	2	DG(i)113-SC/DGi116-Solvent	Tris(hydroxymethyl)-aminomethane

1: Daily standardization 2: Weekly standardization 3: Every two weeks standardization 5: Protect from CO_2 (absorption tube filled with NaOH on carrier granular).

Redox – Titrations

Name	Formula	Solvent	Control	Indication	Standard Substance
Reducing agents					
Ammonium ferrous (II) sulfate	$(NH_4)_2Fe(SO_4)_2$	60 mL 50% H_2SO_4 + deion. H_2O → 1 L	1, 4	DM(i)140-SC, DMi147-SC	Potassium dichromate
Sodium thiosulfate	Na_2S2O_3	deion. H_2O (+ 3 drops $CHCl_3$ + 0.1 g Na_2CO_3)	3	DM(i)140-SC	Potassium iodate
Hydroquinone	$C_6H_6O_2$	20 mL conc. H_2SO_4 + deion. H_2O → 1 L	2, 7	DM(i)140-SC	Potassium dichromate
Oxidizing agents					
Potassium permanganate	$KMnO_4$	deion. H_2O	3, 7	DM(i)140-SC	Di-Sodium oxalate
Iodine	I_2	2.5% KI / deion. H_2O	1, 7, 8, 9	DM(i)140-SC	Di-Sodium oxalate
Cerium (IV) sulfate	$Ce(SO_4)_2$	58 mL 50% H_2SO_40 + deion. H_2O → 1 L	3	DM(i)140-SC	Di-Sodium oxalate
Potassium dichromate	$K_2Cr_2O_7$	deion. H_2O	3	DM(i)140-SC	Di-Sodium oxalate
Iron (III) chloride	$FeCl_3$	deion. H_2O	3	DM(i)140-SC	Ascorbic acid
Sodium nitrite	$NaNO_2$	deion. H_2O	2	DM(i)140-SC	Sulfanilic acid
2,6 Dichlorophenol-indophenol-Na-salt	DPI	deion. H_2O	1, 7, 8, 9	DM(i)140-SC, DP5 Phototrode	Ascorbic acid

1: Daily standardization 2: Weekly standardization 3: Every two weeks standardization 4: Protect from O_2 7: Keep bottle in dark
8: Keep in PE bottles 9: Keep cool

Precipitation Titrations

Name	Formula	Solvent	Control	Indication	Standard Substance
Argentometry					
Silver nitrate	$AgNO_3$	deion. H_2O	3, 7	DM(i)141-SC, DMi148-SC	Sodium chloride
Sodium chloride	NaCl	deion. H_2O	3	DM(i)141–SC	Silver nitrate
Potassium bromide	KBr	deion. H_2O	3	DM(i)141–SC	Silver nitrate
Sulfate/Fluoride					
Barium perchlorate	$Ba(ClO_4)_2$	i-C_3H_7OH/H_2O	2	DP5 Phototrode / with Thorin	Na_2SO_4 solution
Barium chloride	$BaCl_2$	deion. H_2O	2	DP5 Phototrode / with Thorin	Na_2SO_4 solution
Lanthanum nitrate	$La(NO_3)_3$	deion. H_2O	3	DX219 or perfectION Fluoride	NaF solution
Lead nitrate	$Pb(NO_3)_2$	deion. H_2O	3	DX219 or perfectION Fluoride	NaF solution

2: Weekly standardization 3: Every two weeks standardization 7: Keep bottle in dark

Complexometric Titrations

Name	Formula	Solvent	Control *)	Indication	Standard Substance
Complexone III	EDTA	deion. H_2O	3, 8	DP5 Phototrode	Calcium carbonate
Complexone VI	EGTA	deion. H_2O	3, 8	DP5 Phototrode	Calcium carbonate

Turbidimetric Titrations

Name	Formula	Solvent	Control *)	Indication	Standard Substance
N-cetylpyridinium-chloride	CPC	deion. H_2O	3	DP5 Phototrode	Sodium dodecyl sulfate
Sodium dodecyl sulphate	SDS	deion. H_2O	3	DP5 Phototrode	N-cetylpyridinium chloride

Water Titrations

Name	Formula	Solvent	Control *)	Indication	Standard Substance
Karl Fischer reagent		CH_3OH	1, 6	DM143-SC	Di-sodium tartrate dihydrate or deion. H_2O

*) 1: Daily standardization 3: Every two weeks standardization 8: Keep in PE bottles 6: Protect from humidity (fill absorption tube with molecular sieve).

Fundamentals of Titration

This booklet is a comprehensive compendium that covers in detail titration fundamentals such as titration types, indication methods, endpoint or equivalence point recognition and evaluation and more. It shall serve as a tool for (self)-education for titration beginners or as a reference book for advanced titration users that require background information about titration for their specific tasks.

www.mt.com/education-line

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